1. Introduction

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Insurance Economics (LMU, 2024)

Significance of insurance

The **market for insurance is enormous in size**.

Some numbers for Germany (2019)

- 193.9 bn Euro as revenue
- 163.7 bn Euro paid out to cover losses
- 428.8 mn individual contracts
- 529,000 people employed with insurance companies
- 539 insurance companies account for around 4% share of GDP
- German market accounts for approximately 5% of the world market

This does not even account for various other forms and instances where insurance plays a role.

Historical perspective

- Around 1800 BC the *Code of Hammurabi* mentions loans given out for ship voyages that have not to be refunded in case a ship is lost
- The roman emperor Claudius (9 BC - 54 AC) supported corn trade by granting cover for ship losses in storms on the Mediterranean
- In the middle ages occupational guilds provide life rents for families of prematurely deceased members and farmer cooperatives in Italy provide mutual insurance to their members.
- By 1700 the English government and Dutch towns raised funds by issuing annuities (first use of actuarial life expectancy tables)
- By 1720 a market for fire insurance policies emerged in London ($\rightarrow$ Great Fire 1666)
- By 1750 modern system of marine insurances was established in London $\rightarrow$ Lloyd's of London
  - 1687 Lloyd's Coffee House
  - 1696 Lloyd's List
  - 1771 Society of Lloyd's
- 1752 Benjamin Franklin sets up a fire insurance company in the English colonies in North America

What is traded?

- **Obvious answer:** a certain payment, the premium, is exchanged for the promise to pay partial or full compensation (cover) for loss resulting from carefully specified events. So premium is exchanged for cover.
- **Less obvious answer:** state-contingent incomes.
  Income is reduced in states of the world in which the specified events do not happen, and increased in states in which the events do happen, as compared to the situation without insurance.

Methodology

- There are many different insurance markets, corresponding to the types of events concerned.
- We try to **abstract the essential features** of an insurance market and incorporate them into a simple model.
- However, in applications to specific insurance markets it will often be necessary to modify and extend the simple model.

Methodology

- Insurance is an application of **decision making under uncertainty**.
- The first formal studies on how to deal with uncertain events were performed by mathematicians dealing with gambling problems.
- The Italian mathematician Luca Paccioli in 1493 posed the following problem:

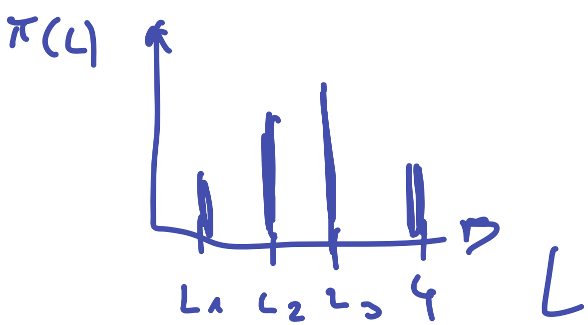
  > *A and B are playing a fair game of balla. They agree to continue until one has won six rounds. The game actually stops when A has won five and B three rounds. How should the stakes be divided.*

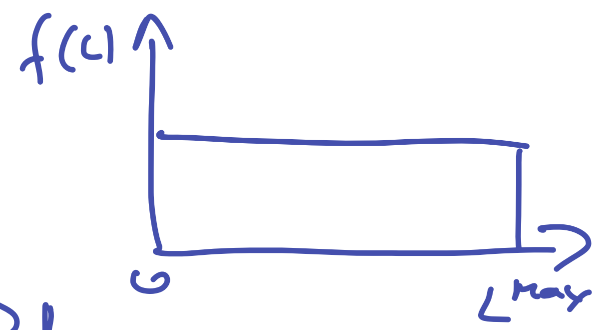
- Mid 17th century: Blaise Pascal and Pierre Fermat are the first to think about probabilities in a modern manner; suggest a solution to the above problem

Methodology

We assume there are **states of the world** which realize with some (exogenous or endogenous) probability. The state of the world is, for our purposes, defined by the amount of loss L suffered.

- **discrete case:** n states $L_1, \ldots, L_n$ which realize with probabilities $\pi_1, \ldots, \pi_n$ (where $\pi_i > 0 \forall i$ and $\sum_{i=1}^{i=n} \pi_i = 1$ has to hold) respectively.
- **continuous case:** A continuum of states $[0, L_{max}]$ where a continuous density function $f(L)$ with $f(L) > 0 \quad \forall \quad L$ and $\int_0^{L_{max}} f(L) \quad dL = 1$ attributes probabilities





Methodology

- In addition assumptions on the **utility function.** We assume the preferences over uncertain alternatives are captured by a Von Neumann and Morgenstern (1944) expected utility function over income y

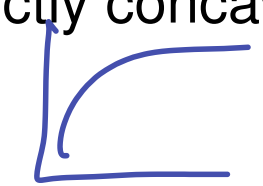
$$EU = \sum_{i=1}^{i=n} \pi_i \quad u(y - L_i)$$

or

$$EU = \int_0^{L_{max}} f(L) \quad u(y - L_i) \quad dL$$

for the continuous case.

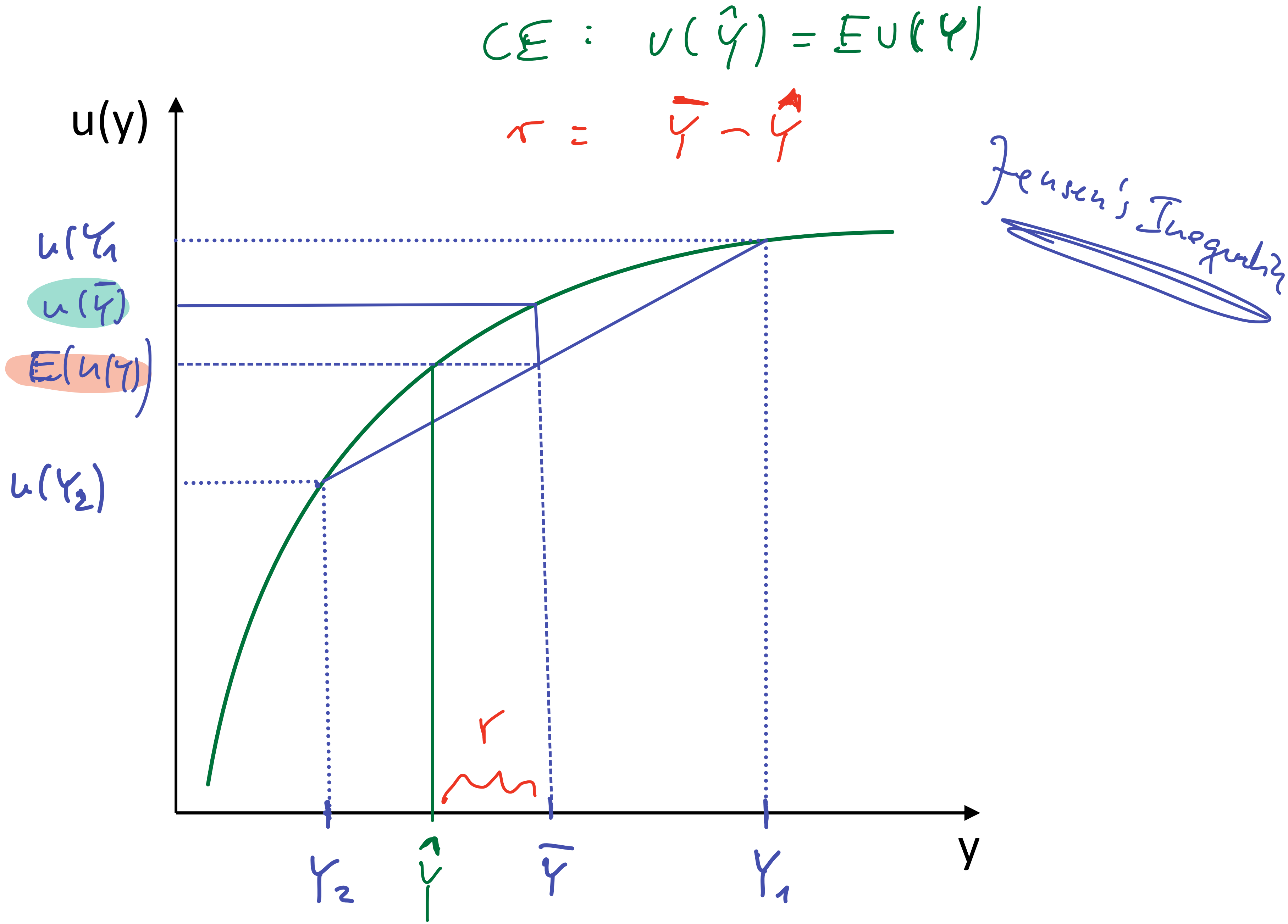
- u_i is uniquely determined up to positive linear transformation
- u_i is assumed to be three times differentiable and strictly concave in income y $u_i^{'}(y) > 0$ and $u_i^{''}(y) < 0$ (Bernoulli 1738)
- Usually we assume state independence of utility



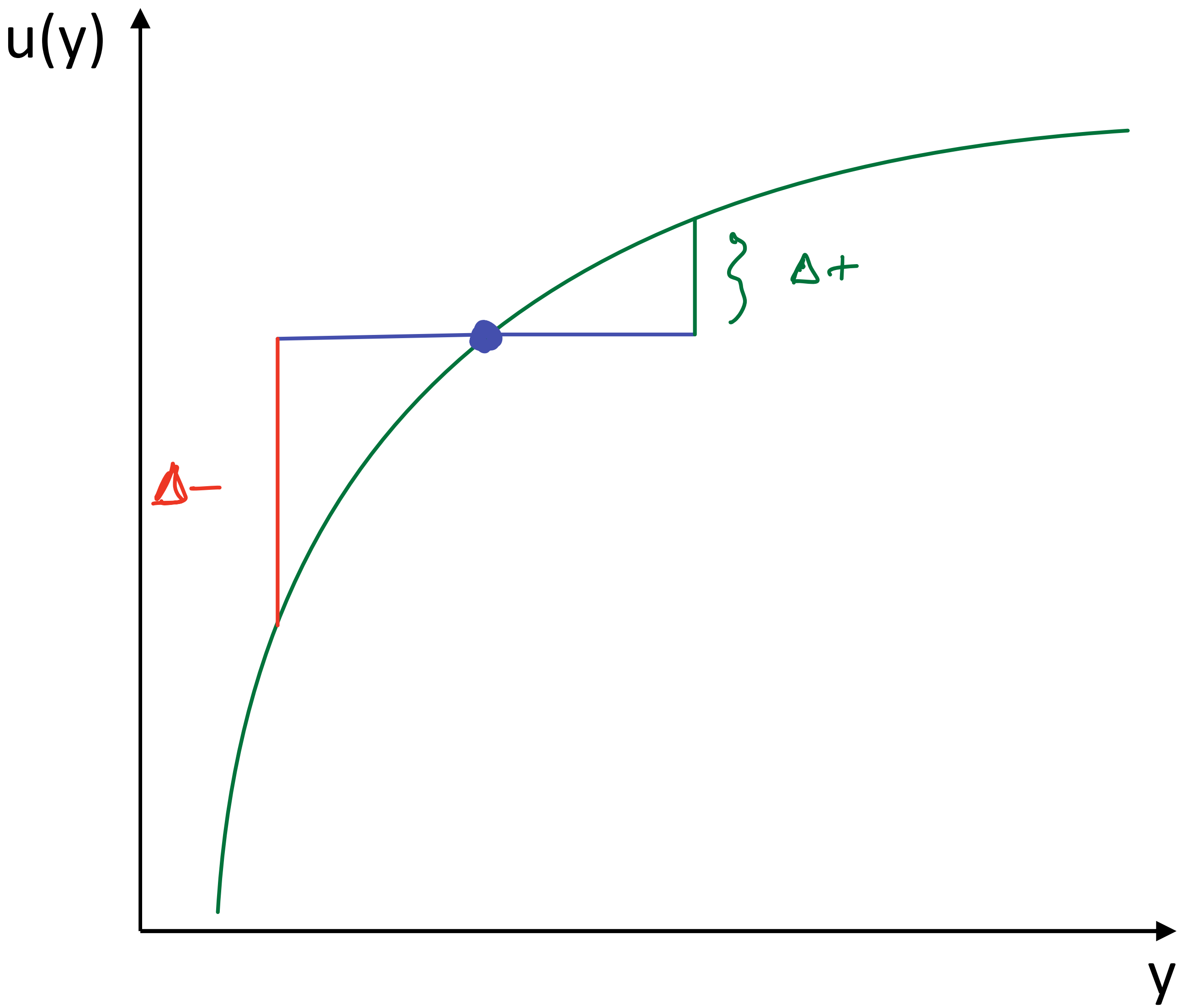
Risk preferences

- It is typically assumed that **humans dislike risk**
- The technical term for this assumption is called **risk aversion**
- There are **several measures** of risk aversion
  - Global measures of RA
  - Local measures of RA
  - Absolute RA
  - Relative RA
  - Partial RA
  - Prudence
- In this course, we will mainly deal with **Absolute Risk Aversion**

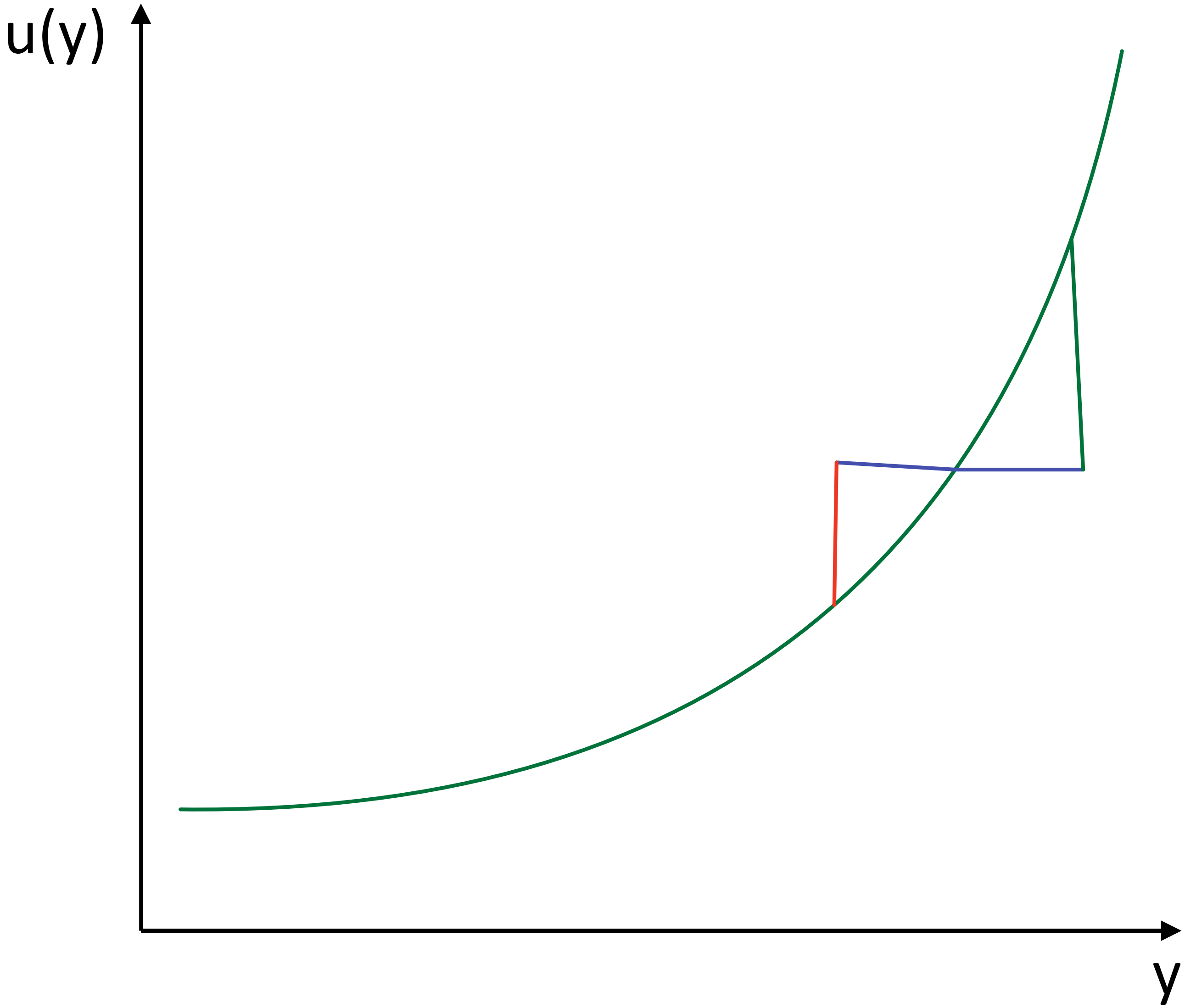
More on risk aversion: Why do we assume it?



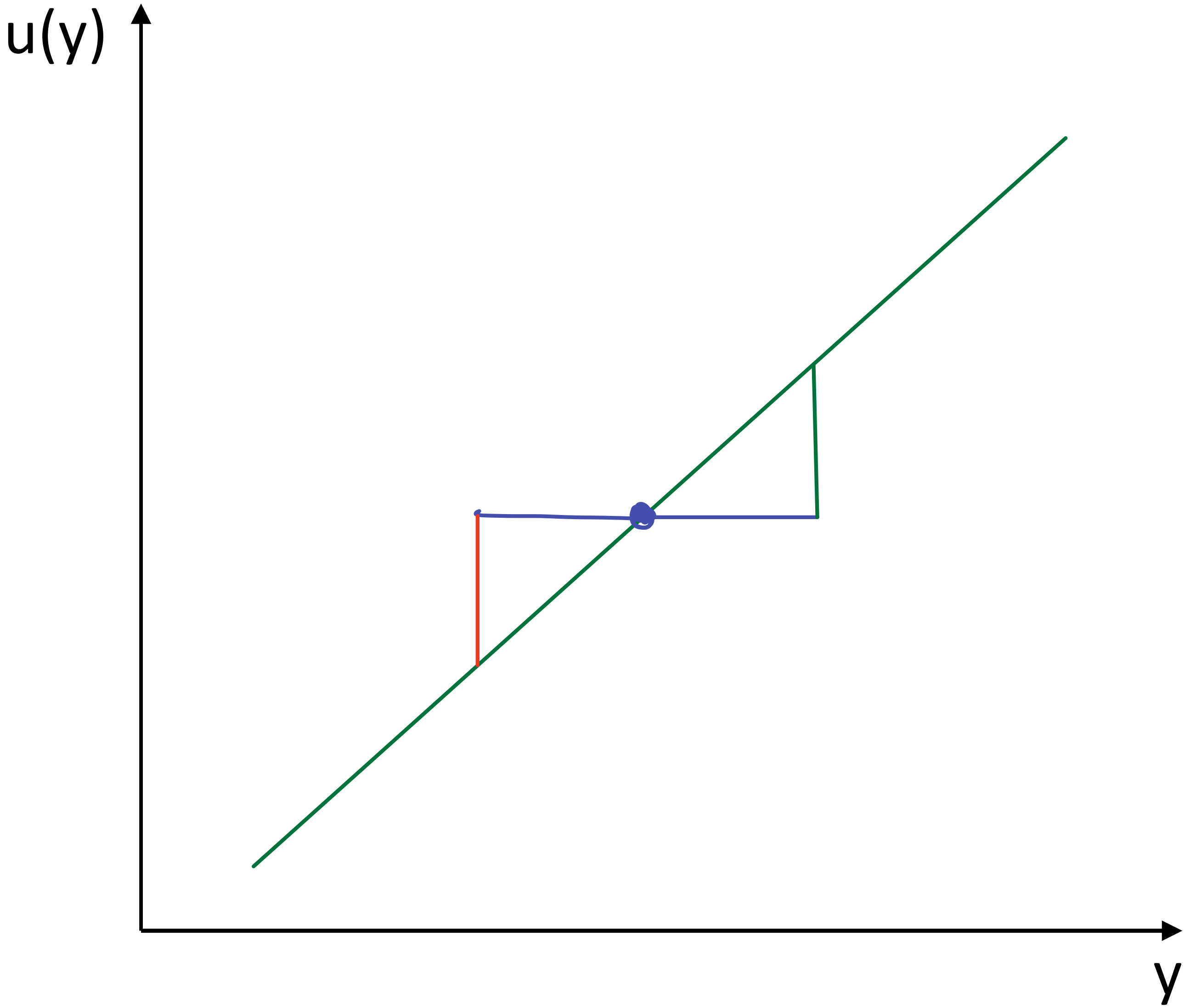
The same story with more intuition



And what about risk affinity (aka risk love) ?



And what about risk neutrality ?



Measure for absolute risk aversion

Assumptions on **risk preferences**

- Arrow Pratt Coefficient of absolute risk aversion

$$A(y) = -\frac{u_i''(y)}{u_i'(y)}.$$

- Used to measure risk attitude. Can be either DARA (Decreasing Absolute Risk Aversion), CARA (Constant Absolute Risk Aversion), or IARA (Increasing Absolute Risk Aversion).
- Under this theory, given a choice set of alternative probability distributions of income, each of which induces a corresponding probability distribution of utilities, the decision taker chooses that distribution with the highest expected value of utility.

Comparative-static properties of $A(y)$

DARA, CARA, and IARA

An at least twice differentiable utility function has the property of ...

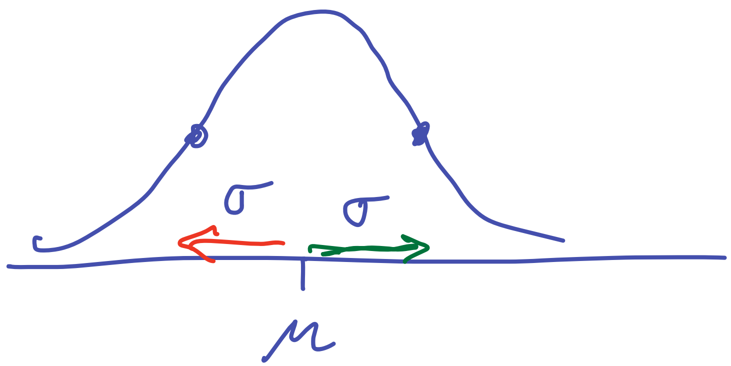
1. ... **decreasing absolute risk aversion (DARA)** iff $\frac{dA(y)}{dy} < 0$.
2. ... **constant absolute risk aversion (CARA)** iff $\frac{dA(y)}{dy} = 0$.
3. ... **increasing absolute risk aversion (IARA)** iff $\frac{dA(y)}{dy} > 0$.

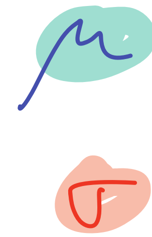
Examples:

1. DARA: The richer I am, the less risk-averse I am to bet 10 EUR.
2. CARA: If I become richer, my risk aversion to bet 10 EUR stays the same.
3. IARA: The richer I am, the more risk-averse I am to bet 10 EUR.

The empirically most realistic assumption is: DARA

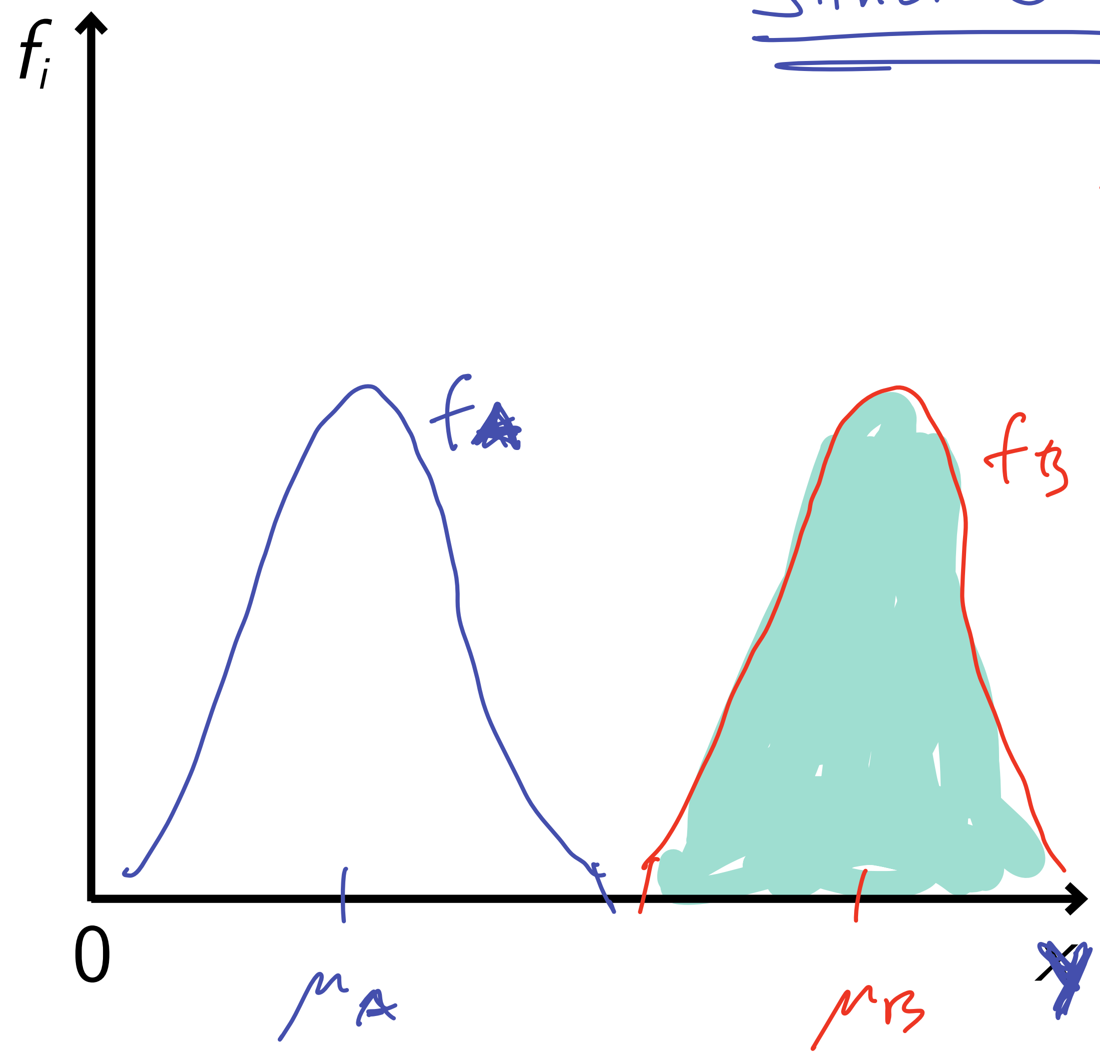
Measuring risk





- We now know how to measure risk aversion
- But **how do we measure risk?**

Example 1



Strict Dominance

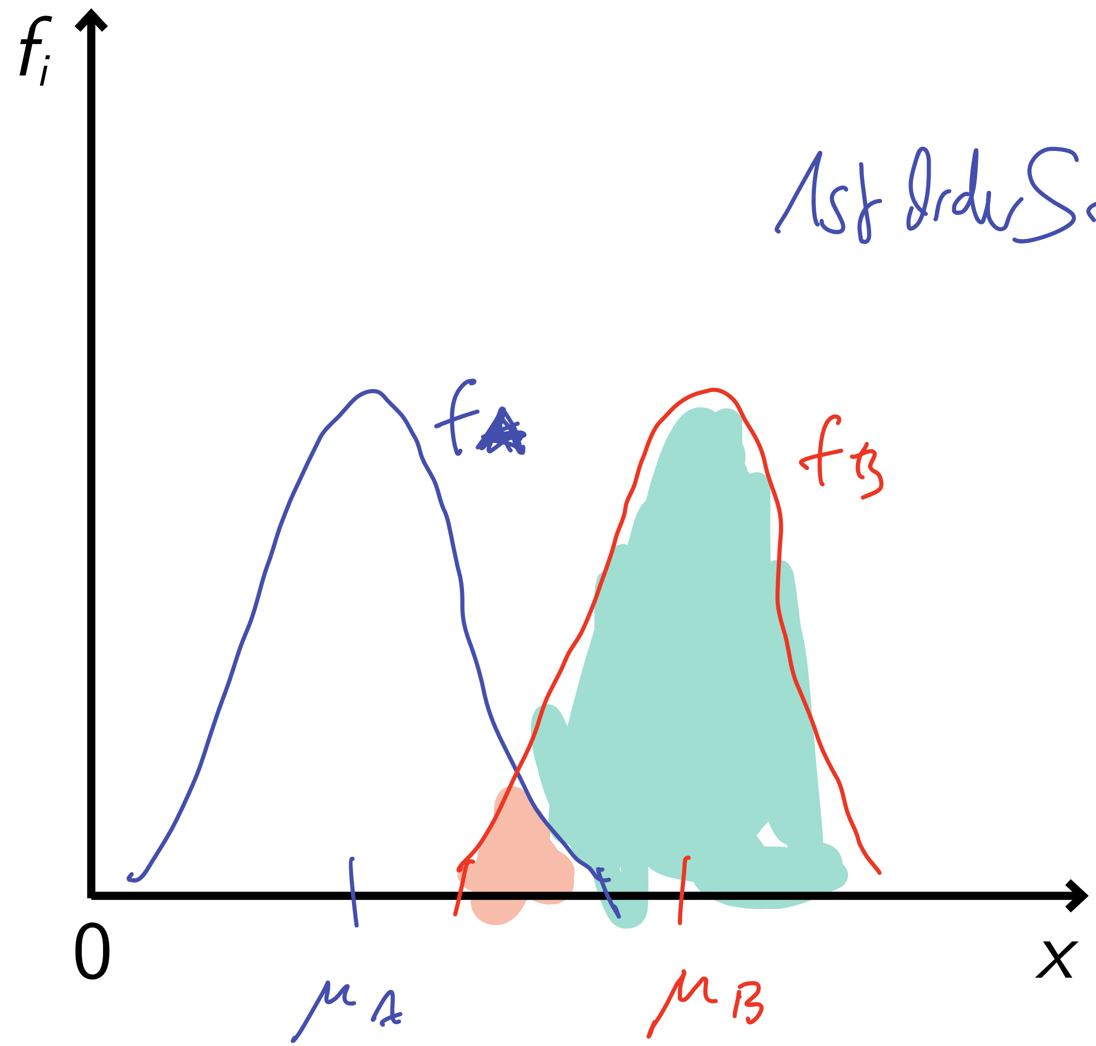
$f_B \succ f_A$

Example 2

$\sigma_A = \sigma_B$

1st Order Stochastic Dominance

$f_B > f_A$

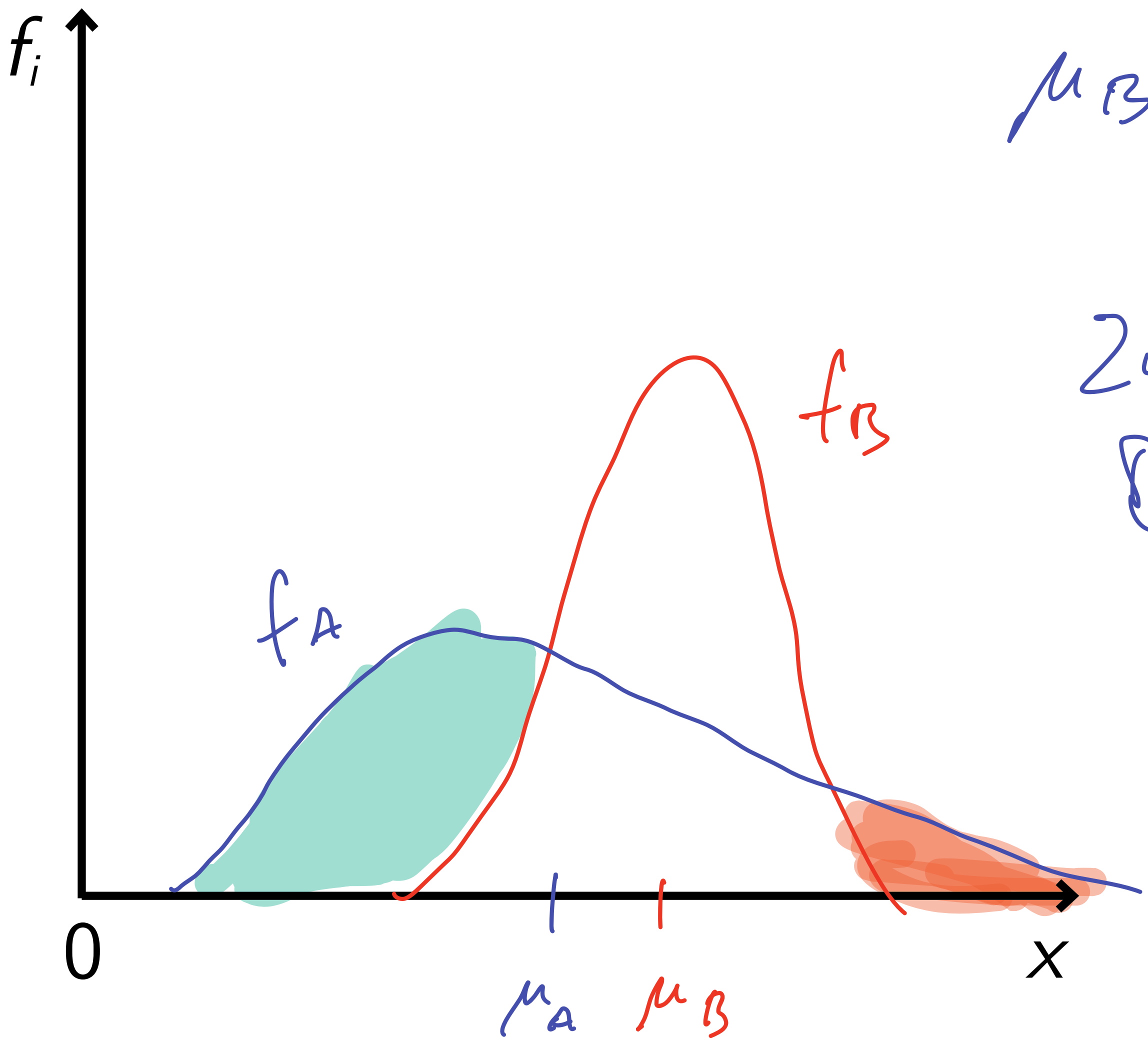


Example 3

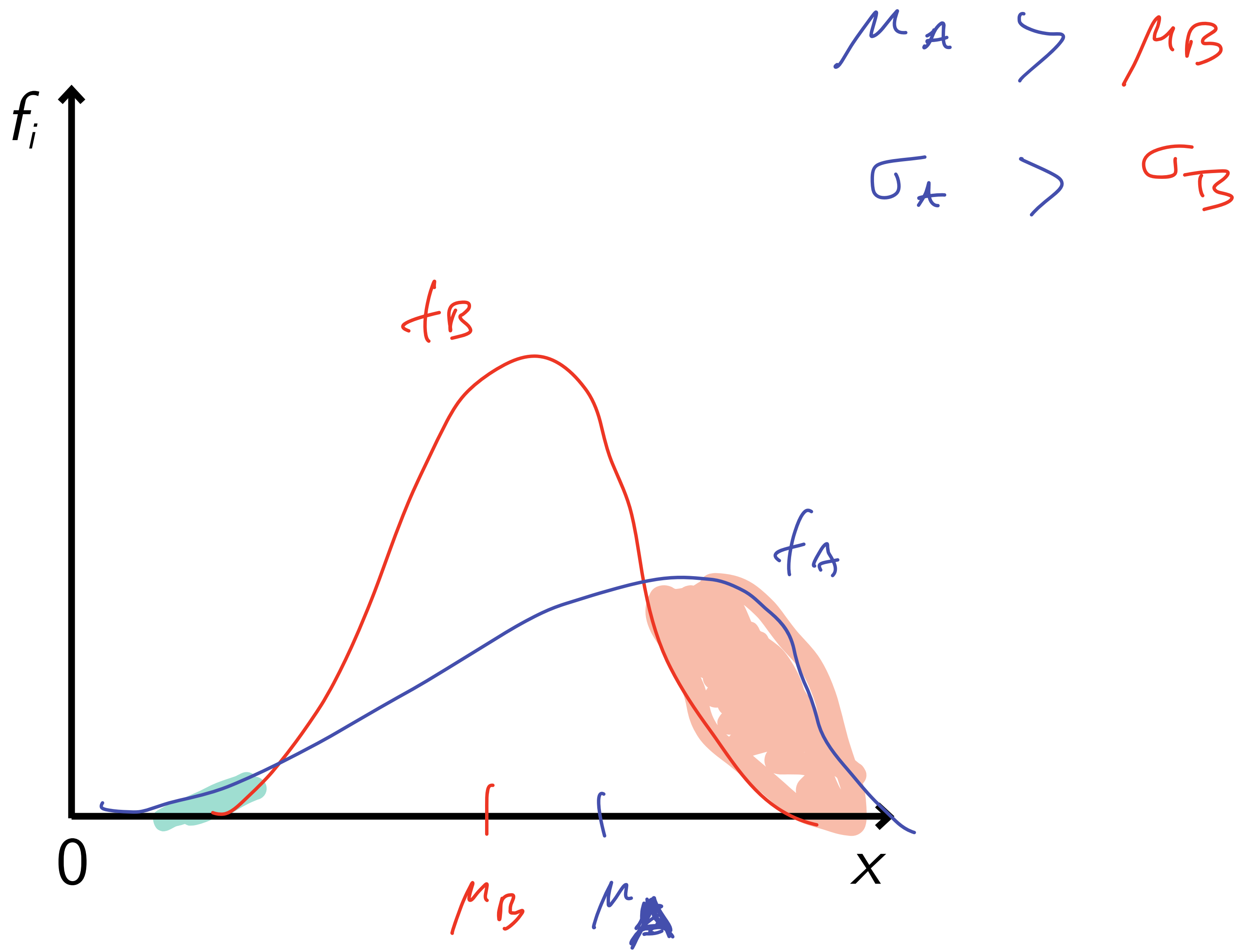
$\sigma_A > \sigma_B$

$\mu_B < \mu_A$

2nd order Stochast Dominance



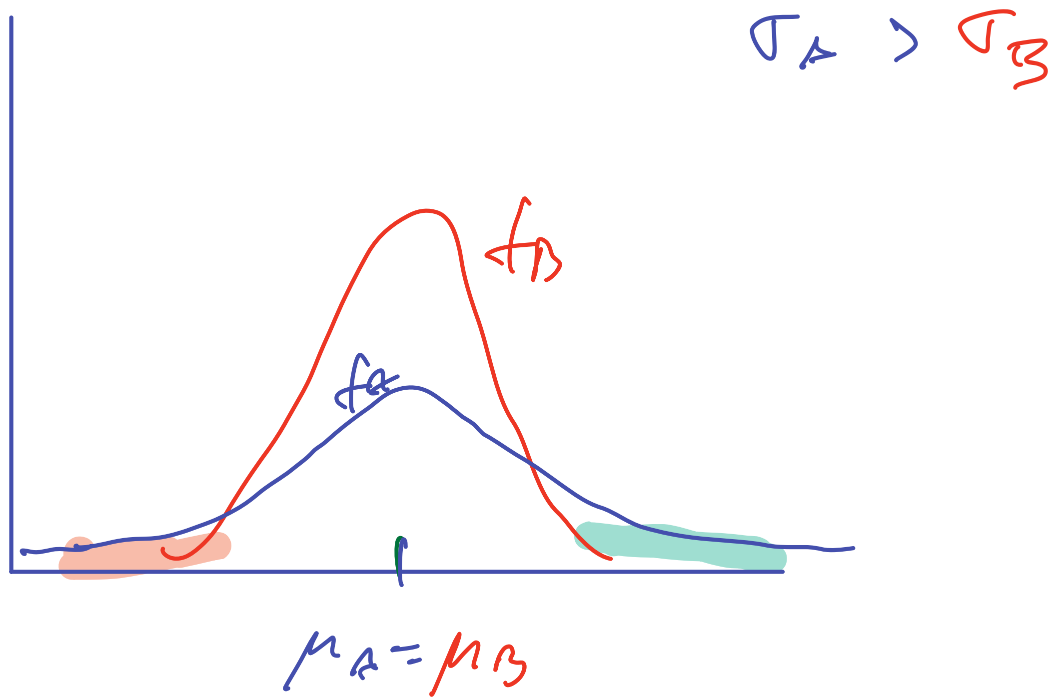
Example 4



Stochastic dominance

Mean - preserving spread

- The most common concept to quantify risk is called **stochastic dominance**
- See handout



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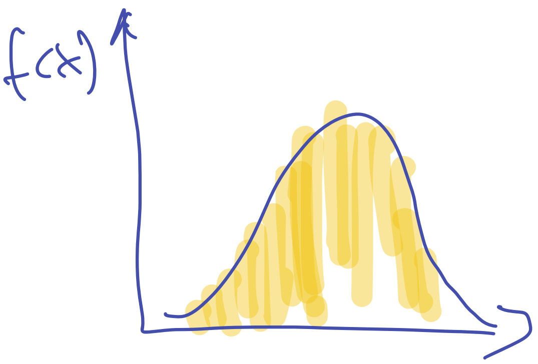

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