2. Insurance Demand

Till Stowasser

University of Stirling

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Two basic models

$q^*(P, Y, L, \pi)$

- *The q– Model* A certain payment, the **premium**, is exchanged for the promise to pay partial or full compensation (**cover**) for **loss** resulting from carefully specified **loss events**.

y_1^* y_2^*

- *The y– Model* Income is reduced in states of the world in which the specified events do not happen, and increased in states in which the events do happen, as compared to the situation without insurance. → Insurance as state-contingent income

The two models are **fully equivalent**. The *q*-model is often easier to handle mathematically. The advantage of the *y*-model is that it lends itself to a diagrammatical treatment and exploits the similarities with standard consumer theory.

Simplest possible model: 2 states (1, 2)

Basic setup for both models

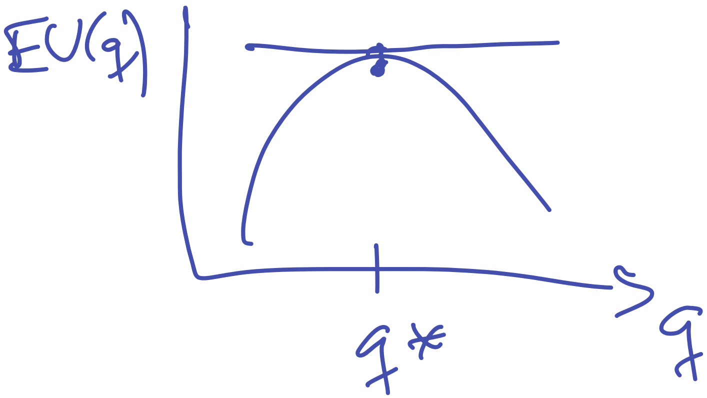
- income in the no-loss state: $y_1 = y$
- income in the loss state: $y_2 = y - L$
- probability of loss L is π
- expected value of income without insurance:

$$\bar{y} = (1-\pi)y + \pi(y-L) = y - \pi L$$

- expected value of income loss: πL
- expected utility in the absence of insurance:

$$EU = (1-\pi)u(y) + \pi u(y-L)$$

- In the absence of insurance the individual has an uncertain income endowment.



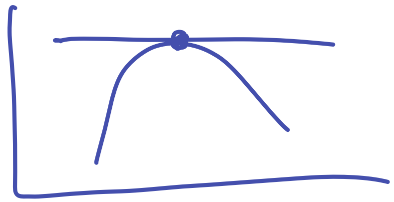
The q–Model

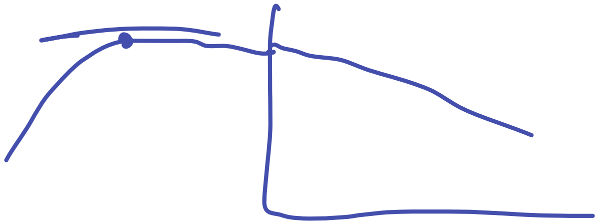
- Insurer offers cover q at a *premium rate p*, where p is a real number (as is a probability). The *premium amount P* (Euro) is pq.
- Note that assuming the marginal price of cover p to be constant is restrictive in the sense that it rules out convex price schemes.
- The buyer chooses $q \geq 0$ to maximize

$$\max_q \; EU(q) = (1-\pi)u(\underbrace{y - pq}_{y_1}) + \pi u(\underbrace{y - L + (1-p)q}_{y_2}).$$

$$\frac{\partial EU(q)}{\partial q} = \underbrace{-p}_{\partial y_1/\partial q}\underbrace{(1-\pi)\cdot u'(y_1)}_{\partial EU/\partial y_1} + (1-p)\pi \cdot u'(y_2) = 0$$

First-order conditions





Standard first order (Kuhn-Tucker) condition

$$
\begin{aligned}
EU_q &= -p(1-\pi)u'(\overbrace{y-pq^*}^{Y_1 = Y_G}) + (1-p)\pi u'(\overbrace{y-L+(1-p)q^*}^{Y_2 = Y_B}) \leq 0 \\
q^* &\geq 0 \\
EU_q q^* &= 0
\end{aligned}
$$

$E(MC) = E(MB)$

Note that the second order condition is, as required for an optimum, globally negative.

First Order Conditions

There are 2 cases:

$\frac{\partial EU}{\partial q} = 0$ $\boxed{q^* = L}$

- Either:

$$q^* > 0 \Rightarrow \frac{1-p}{p} = \frac{1-\pi}{\pi} \frac{u'(y - pq^*)}{u'(y - L + (1-p)q^*)} \tag{1}$$

$\frac{\partial EU}{\partial q} < 0 \Rightarrow q \overset{!}{=} 0$

- Or:

$$\frac{1-p}{p} < \frac{1-\pi}{\pi} \frac{u'(y - pq^*)}{u'(y - L + (1-p)q^*)} \Rightarrow q^* = 0$$

- The latter condition is equivalent to saying that the demand for cover will be only positive if the price p is “not too high”, namely

$$p < \frac{\pi u^{'}(y - L + (1-p)q^*)}{(1-\pi)u'(y - pq^*) + \pi u'(y - L + (1-p)q^*)}.$$

First-order conditions

From now on we will generally neglect the Kuhn–Tucker conditions, implicitly focussing on the economically more interesting case of an **interior solution**. Assuming $q^* > 0$ and rearranging the first order condition (1)

$$EU_q = -p(1-\pi)u^{'}(y - pq^*) + (1-p)\pi u^{'}(y - L + (1-p)q^*) = 0$$

$$q^* > 0$$

the following must hold:

- $p = \pi \Leftrightarrow q^* = L$
  with a *fair premium* there is *full cover*
- $p > \pi \Leftrightarrow q^* < L$
  with a *positive loading* there is *partial cover*
- $p < \pi \Leftrightarrow q^* > L$
  with a *negative loading* there is *more than full cover.*

?

First-order conditions

In particular it holds that

$$
\begin{aligned}
p &= \pi \\
&\Leftrightarrow \\
u^{'}(y - pq^{*}) &= u^{'}(y - L + (1 - p)q^{*}) \\
&\Leftrightarrow \\
q^{*} &= L.
\end{aligned}
$$

Comparative statics

- Explore the **relationships between the optimal value** of the endogenous variable, the demand for insurance, **and the exogenous variables** that determine it, p, π, L, y.
- **Note:** If we assume a fair premium, full cover is always bought $\Rightarrow$ comparative statics analysis is trivial.
- Assume (realistically) that $p > \pi$ and thus $0 < q^* < L$
- First Order Condition is

$$EU_q = -p(1-\pi)u^{'}(y - pq^*) + (1-p)\pi u^{'}(y - L + (1-p)q^*) = 0.$$

Comparative statics

Applying standard methods of comparative statics (**implicit function theorem**) we have that

$$
\begin{aligned}
(1) \quad & \frac{\partial q^*}{\partial y} = -\frac{EU_{qy}}{EU_{qq}} < 0 \quad \checkmark \\
(2) \quad & \frac{\partial q^*}{\partial L} = -\frac{EU_{qL}}{EU_{qq}} > 0 \\
(3) \quad & \frac{\partial q^*}{\partial p} = -\frac{EU_{qp}}{EU_{qq}} < 0 \\
(4) \quad & \frac{\partial q^*}{\partial \pi} = -\frac{EU_{q\pi}}{EU_{qq}} > 0
\end{aligned}
$$

Because of risk aversion ($u'' < 0$) it is easy to show that $EU_{qq} < 0$

$\Rightarrow$ sign of derivatives is determined by that of the numerator

Comparative statics

$$EU_q = -p(1-\pi)u'(y - pq^*) + (1-p)\pi u'(y - L + (1-p)q^*) = 0.$$

$\hookrightarrow p(1-\pi) = (1-p)\pi \cdot \frac{u'(y_2)}{u'(y_1)}$

- Let's start with an **increase in income** y:

$$EU_{qy} = -p(1-\pi)u''(y - pq^*) + (1-p)\pi u''(y - L + (1-p)q^*) \leq / \geq 0$$

- Indeterminacy of the sign of this effect is no surprise: As in standard consumer theory income effects can go either way and insurance cover can be an inferior or a normal good.
- **Relate this to the buyer's risk preferences:**

$$y_1^* \equiv y - pq^* \qquad \text{and} \qquad y_2^* \equiv y - L + (1-p)q^*$$

are the optimal incomes in the two states.

$A = -\frac{u''}{u'}$

- We know that $y_1^* > y_2^*$ (because of partial cover).

Comparative statics

Rearranging the FOC gives   Rewritten FOC

$$\boxed{p(1-\pi) = (1-p)\pi \frac{u'(y_2^*)}{u'(y_1^*)}}$$

Substituting gives

$$
\begin{aligned}
EU_{qy} &= -u''(y_1^*)\frac{(1-p)\pi u'(y_2^*)}{u'(y_1^*)} + (1-p)\pi u''(y_2^*) \\
&= EU_{qy} = (1-p)\pi \underline{u'(y_2^*)}\left[\frac{u''(y_2^*)}{u'(y_2^*)} - \frac{u''(y_1^*)}{u'(y_1^*)}\right]
\end{aligned}
$$

Factor out $u'(y_2)$

$-A(y_2) + A(y_1)$

Recall the Arrow-Pratt measure of (absolute) risk aversion: $A(y) \equiv -\frac{u''(y)}{u'(y)}$.

Comparative statics

We can then write

$$\frac{\partial q^*}{\partial y} = - \frac{\boxed{EU_{qy}}}{EU_{qq}} \qquad y_1^* \geq y_2^*$$

$$EU_{qy} = (1-p)\pi u'(y_2^*)[A(y_1^*) - A(y_2^*)] \quad < 0$$

if DARA

Thus

$$EU_{qy} \geq / \leq 0 \quad \Leftrightarrow \quad A(y_1^*) \geq / \leq A(y_2^*)$$

- **Insurance cover is a normal good** if risk aversion increases with income (IARA, $A(y_1^*) > A(y_2^*)$ with $y_1^* > y_2^*$).
- **Insurance cover is an inferior good** if risk aversion decreases (DARA, $A(y_1^*) < A(y_2^*)$ with $y_1^* > y_2^*$)
- **Straightforward intuition:** if an increase in income increases one's willingness to bear risk, then one's demand for insurance falls.
- What does this mean for realistic risk-preference assumptions?

Comparative statics

Next we analyze the effect of an **increase in the loss** L

$$EU_{qL} = -(1-p)\pi u^{''}(y_2^*) > 0$$

Thus, as we would intuitively expect, an increase in loss increases the demand for cover, *other things being equal*.

Comparative statics

Thirdly we analyze the effect of an **increase of the premium rate**

$$EU_{qp} = -[(1-\pi)u^{'}(y_1^*) + \pi u^{'}(y_2^*)] + [p(1-\pi)u^{''}(y_1^*) - (1-p)\pi u^{''}(y_2^*)]q^*.$$

But notice that the second term is just $-EU_{qy}q^*$. In fact we have a standard Slutsky equation, which we can write as

$$\frac{\partial q^*}{\partial p} = -\frac{EU_{qp}}{EU_{qq}} = \frac{(1-\pi)u^{'}(y_1^*) + \pi u^{'}(y_2^*)}{EU_{qq}} + q^* \frac{EU_{qy}}{EU_{qq}}$$

The first term is the substitution effect, and is certainly negative ($EU_{qq} < 0$). The second term is the income effect and can be positive or negative (or zero).

SE < 0 IE > 0

Comparative statics

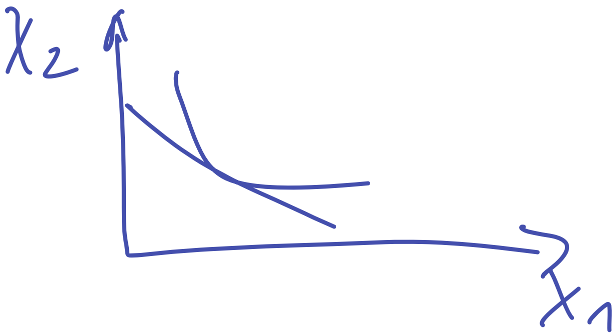
- If **insurance is a normal good** it holds that $EU_{qy} > 0$, hence the second term (the income effect) is negative and there is no ambiguity for IARA. For CARA, $EU_{qy} = 0$ and the second term is zero. Hence, there is no ambiguity, too. I.e., the demand for cover certainly falls as the premium rate (price) rises.
- If **insurance is an inferior good** it holds that $EU_{qy} < 0$ (DARA) and hence the second term is positive and so works against the substitution effect. That is, insurance may be a Giffen good if risk aversion decreases sufficiently with income.
- **Intuition:** An increase in the premium rate increases the price of income in state 2 relative to that in state 1, and so, with utility held constant, y_1 will be substituted for y_2, implying a reduced demand for cover. However, the *increase in premium also reduces real income*, to an extent dependent on the amount of cover already bought, q^*, and this will tend to increase the demand for insurance if DARA, and decrease it if IARA.

Comparative statics

- Finally we look at the effect of an **increase in the loss probability**.

$$EU_{q\pi} = pu^{'}(y_1^*) + (1 - p)u^{'}(y_2^*) > 0$$

- Thus, as we would expect, an increase in the risk of loss increases demand for cover.
- **Note:** There is a strong "other things equal" assumption here. In general we would not expect the premium to remain constant when the loss probability changes.
- We need some theory of the supply side of the market before we can predict how it would change. Thus the above does not give the full market comparative statics of a change in loss probability. Exactly the same point applies to the change in L.



The y–Model

Let the choice variables in the problem be y_1 and y_2 respectively. The buyer's objective is

$$\max_{y_1 y_2} EU = (1-\pi)u(y_1) + \pi u(y_2).$$

Define the budget constraint:

$$q = \frac{y - y_1}{p}$$

$$y_1 = y - pq \quad (2)$$
$$y_2 = y - L + (1-p)q \quad (3)$$

The y–Model

Solving for q in the first equation, substituting into the second and rearranging gives

$$(1-p)y_1 + py_2 = y - pL.$$

$\hat{=}\quad p_1 x_1 \quad p_2 x_2 = m$

Budget constraint:

- $(1-p)$ price of y_1; p the price of y_2
- $y - pL$ "income", a constant, given p

Interpretation:

- with $q > 0$ the buyer leaves the initial endowment point $(y, y-L)$
- if there are no constraints on q, all points satisfying the constraint, including the certain income, are attainable
- the rate of exchange of the state contingent incomes is $-(1-p)/p$
- the demand for cover can be interpreted as the demand for income in the loss state

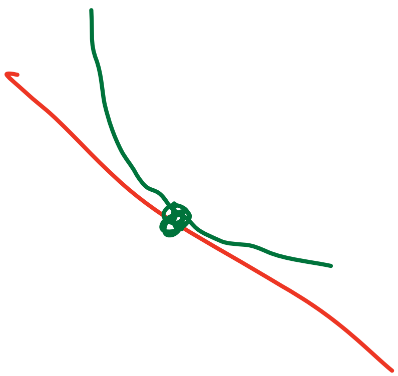
Diagrammatical interpretation

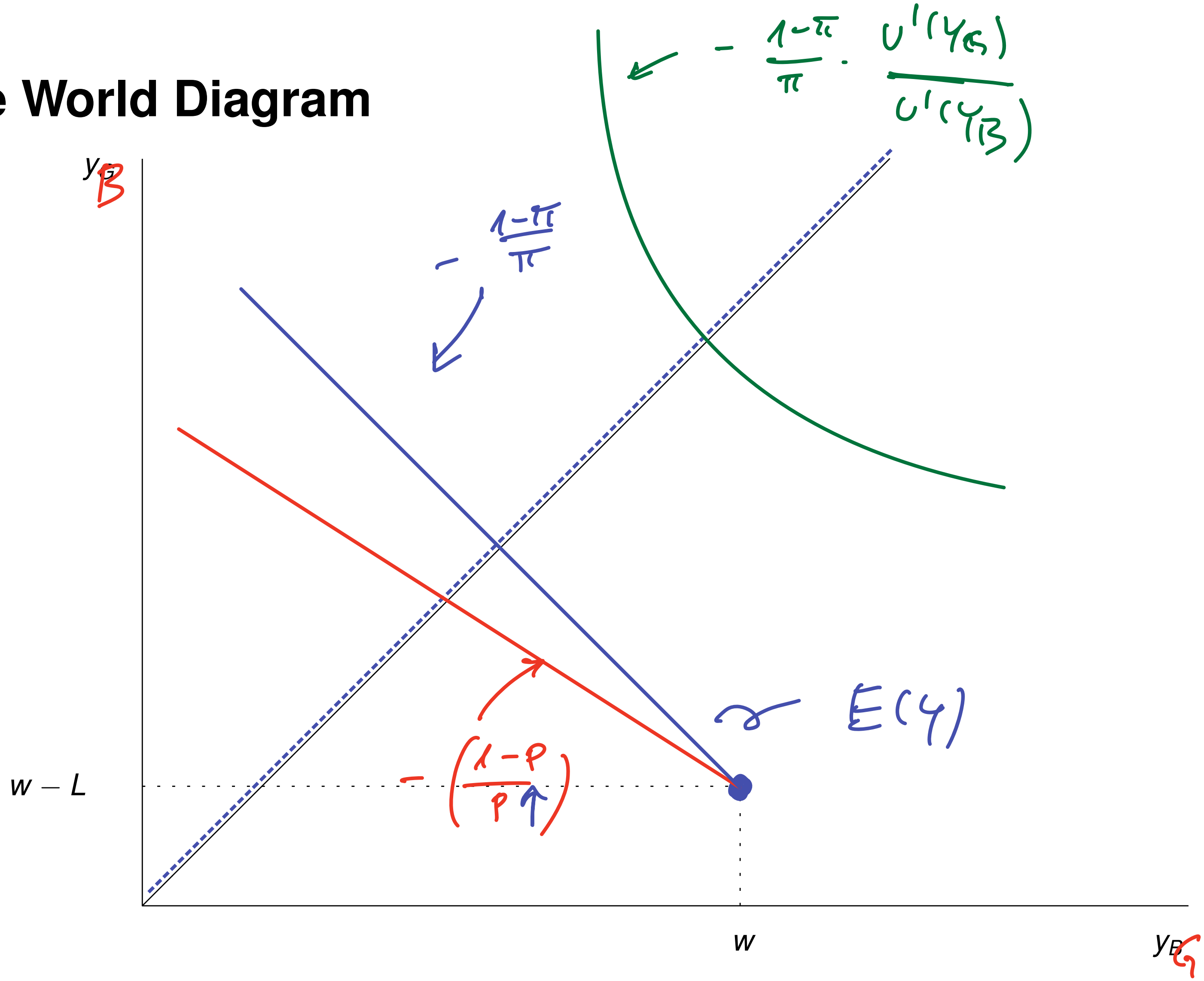
- The endowment point $y_1 = y$, $y_2 = y - L$ satisfies the budget constraint.
- The constraint is a line with slope $-(1-p)/p$ passing through the point $(y, y - L)$.
- Define the *expected value line* by

$$(1-\pi)y_1 + \pi y_2 = \bar{y}.$$

- This line also passes through the initial endowment point $(y, y - L)$, with slope $-(1-\pi)/\pi$.
- Any indifference curve, representing a given expected utility in (y_1, y_2) -space, has a slope of $-(1-\pi)/\pi$ at the point at which it cuts the *certainty line*.

Two States of the World Diagram





Diagrammatical interpretation

The demand for insurance – Full insurance

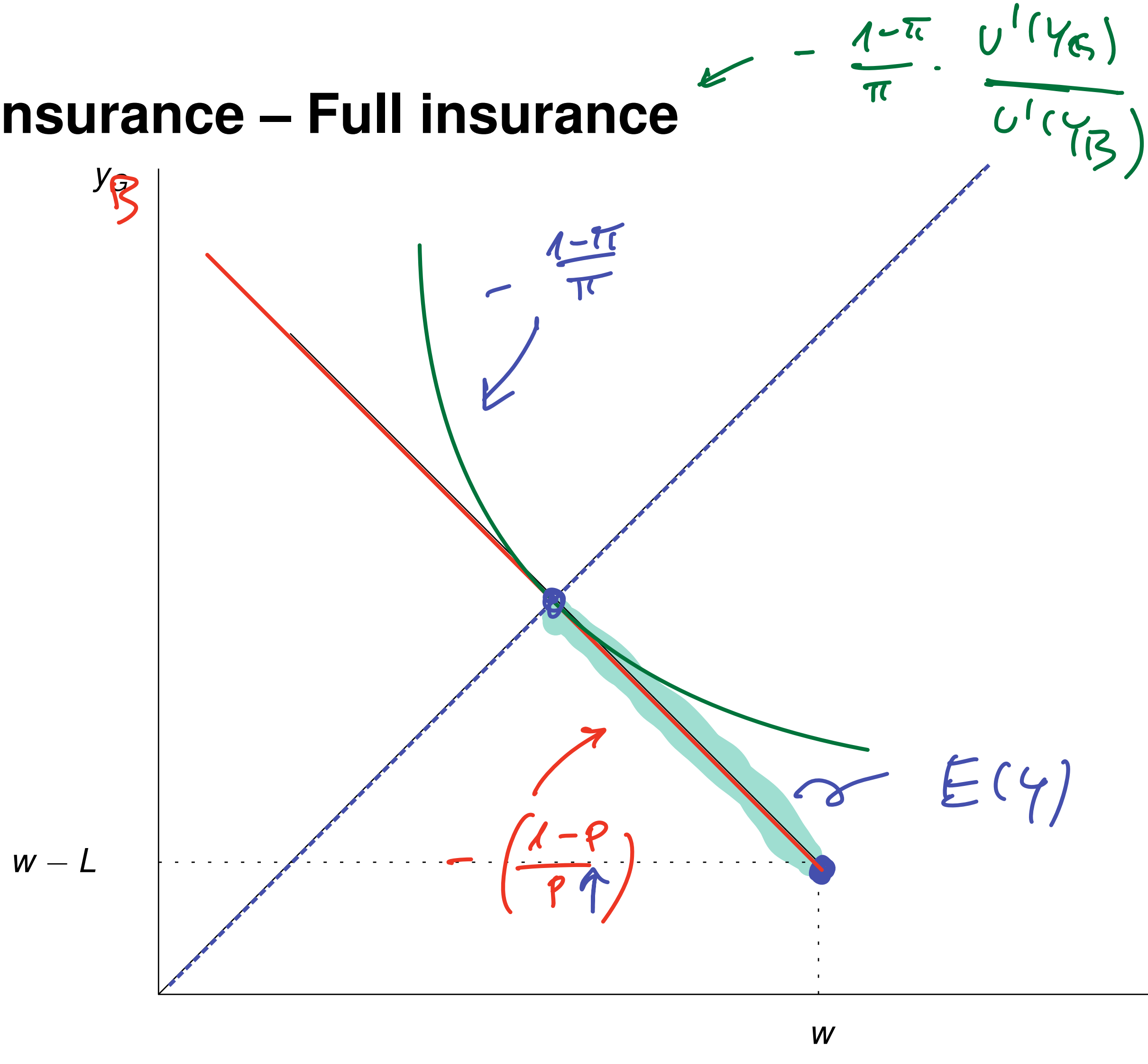
$\pi = p$

On the 45° line, the slope of any indifference curve

$-\frac{1-\pi}{\pi} \cdot \cancel{\frac{u'(y_G)}{u'(y_B)}}$

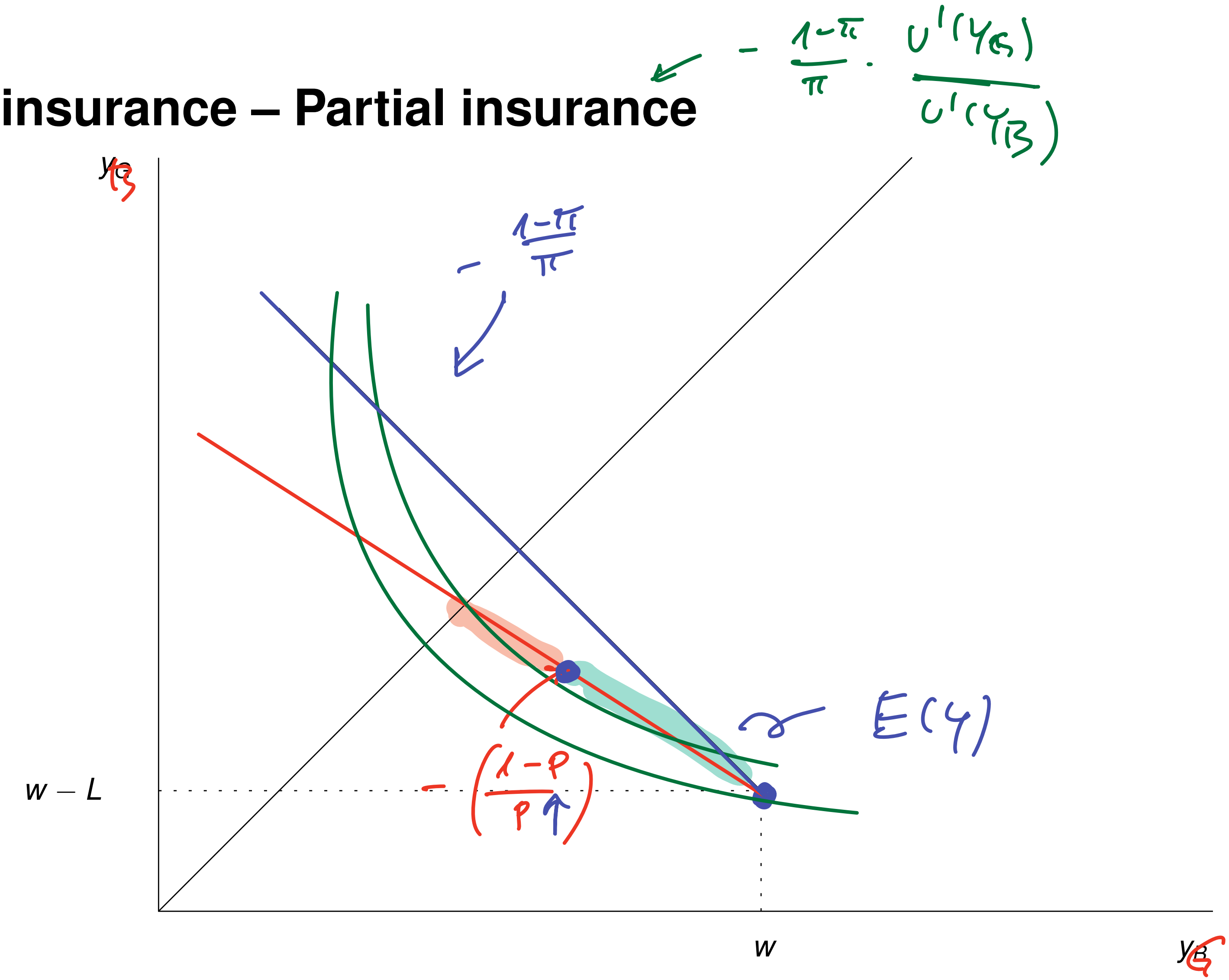
$y_B = y_G$

$-\frac{1-\pi}{\pi}$

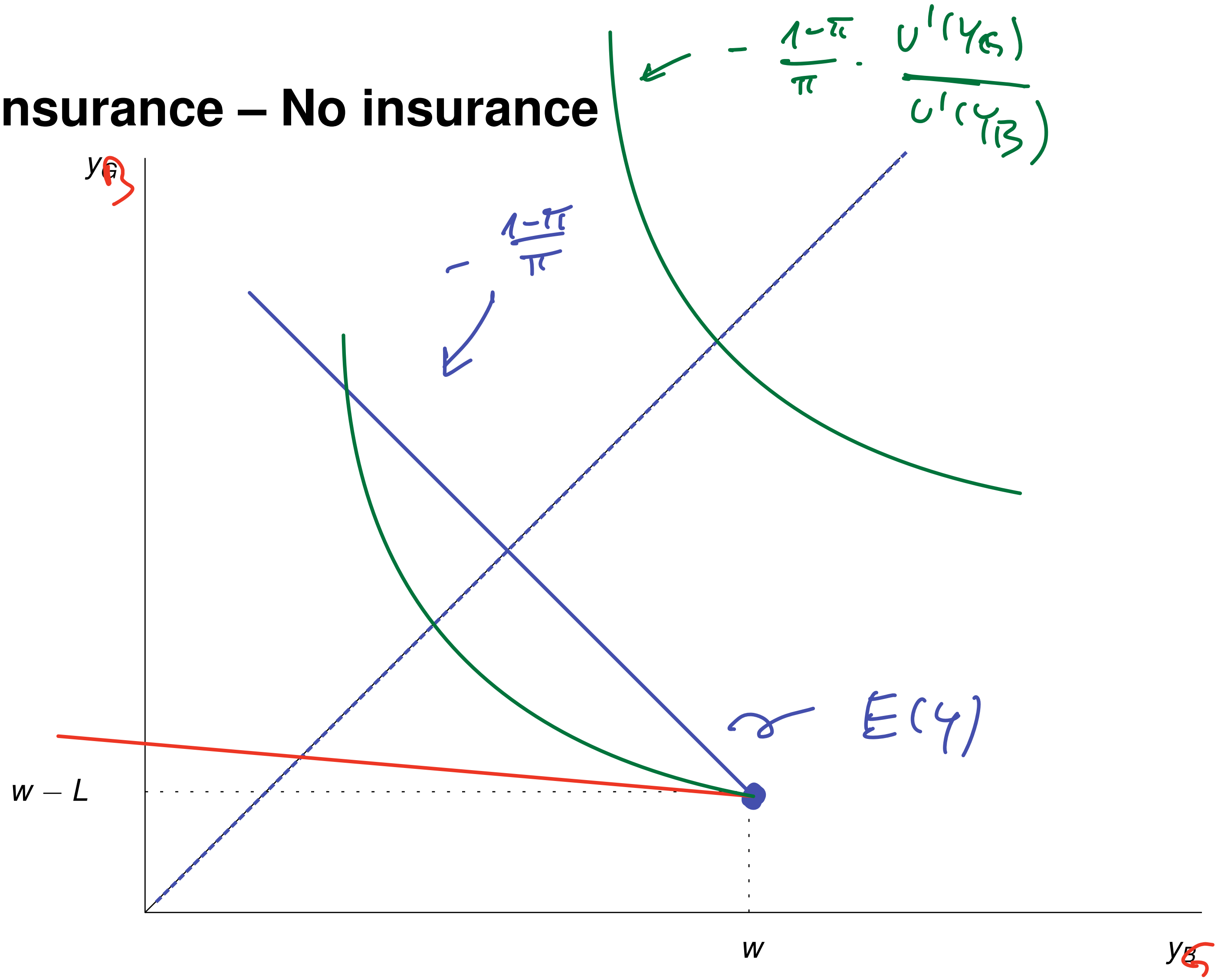


The demand for insurance – Partial insurance

$p = \pi$

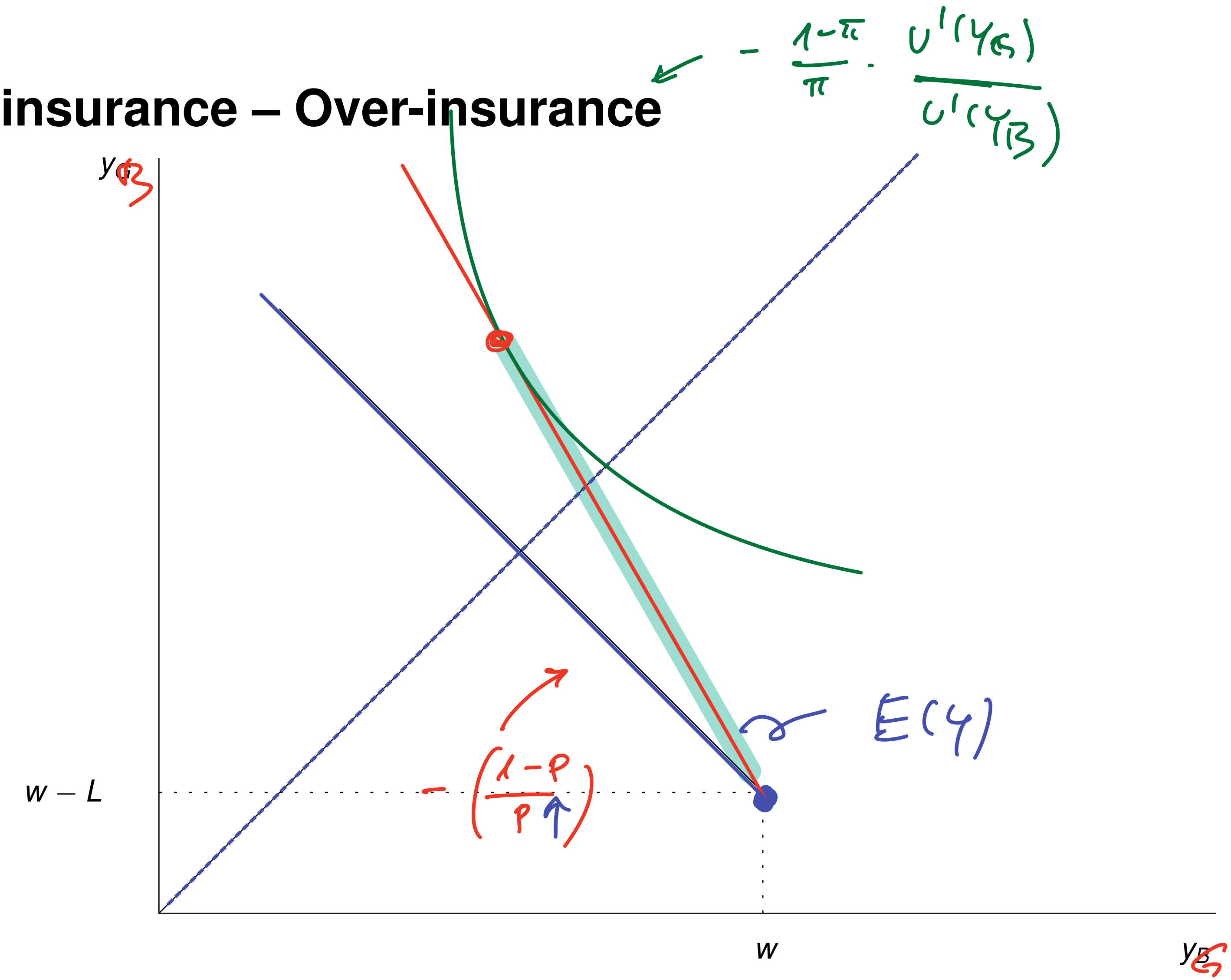


The demand for insurance – No insurance



Diagrammatical interpretation

The demand for insurance – Over-insurance



Diagrammatical interpretation

- The coverage chosen is always at a point of tangency between an indifference curve and a budget line, for $q^* > 0$. Thus the cases of **full**, **partial** and **more than full cover** correspond to the cases in which the budget constraint defined by p is respectively **coincident** with, **flatter** than, or **steeper** than the expected value line.
- **Note:** If the budget line is so flat that it does not intersect the indifference curve passing through the initial endowment point, then we have $q^* = 0$.
- **Note:** If only full or zero cover are available, the buyer takes full cover if and only if the resulting point on the certainty line is above the *certainty equivalent* of the initial endowment point at $\tilde{y}$.

Summary

- We have in fact two models to discuss the demand for insurance.
- The q**-model** derives optimal *cover* as a function of the parameters of the problem

$$q^* = q(p, \pi, L, y)$$

- The $y-$**model** derives the desired *state contingent incomes* as functions of the parameters of the problem

$$y_s^* = y_s(p, \pi, L, y) \quad s = 1, 2$$

- $q-$**model** more suitable for an algebraic treatment
- $y-$**model** more suitable for a diagrammatic treatment

Multiple loss states

- straightforward to extend the model to multiple loss states / continuous losses
- cover as function of loss
- deductible vs. coinsurance

Incomplete markets

- individual may be exposed to several risks
- some of them may not be insurable (e.g. income fluctuations due to the business cycle, ...)
- uninsurable risks affect demand for cover against insurable losses
- effect on demand depends on correlation of risks assume fair premium (otherwise dependent on CARA/DARA/IARA)
    - **positive correlation**
      demand for cover increased – overinsure in order to insure against non–insurable risk
    - **independence**
      demand for cover unchanged
    - **negative correlation**
      demand for cover decreased – the two risks help to smooth income without insurance

State-dependent utility

- For some types of losses for which insurance can be bought, the utility of income will depend on whether or not a particular event takes place, where this event may or may not also cause an income loss.
- **Obvious example:** Sickness – utility of income if one is sick may well differ from that if one is healthy

A simple model

- state 1 is the no-loss state and state 2 is the loss-state
- denote the utility function in state $s = 1, 2$ as $u_s(y)$
- utilities may differ in **absolute terms**, $u_1(y) > u_2(y) \quad \forall \quad y > 0$ or in **marginal terms**, $u_1'(y) > u_2'(y) \quad \forall \quad y > 0$.
- linear example: $u_1(y) = a + bu_2(y) \quad$ with $\quad a > 0 \quad$ and $\quad b > 1$
  Note: assumptions on a and b are unnecessarily restrictive
- otherwise standard von Neumann-Morgenstern utility functions
- for simplicity assume insurance is offered at a fair premium $\quad \rightarrow \quad p$ therefore denotes both the probability of loss and the premium rate
- using the y-model, we have to solve

$$\boxed{\max_{y_1 y_2}(1-p)u_1(y_1) + pu_2(y_2) \quad \text{s.t.} \quad (1-p)y_1 + py_2 = \bar{y}}$$

where $\bar{y}$ is the expected value of income

FOC : $MU_1 = MU_2$

A simple model

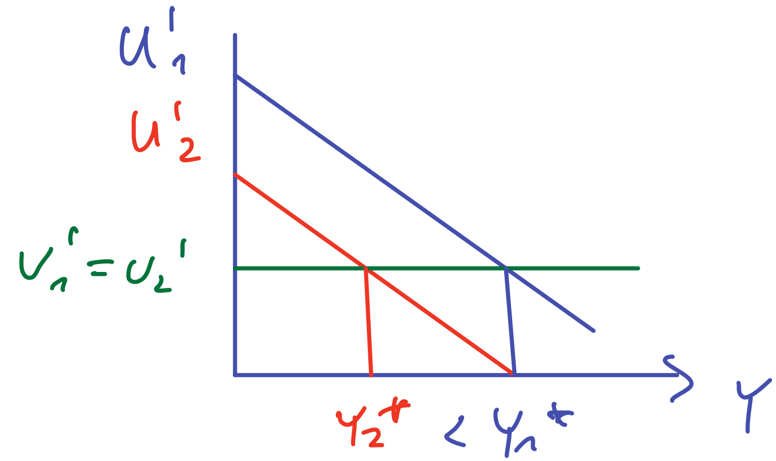
- Assuming an interior solution the optimum requires

$$u_1'(y_1^*) = u_2'(y_2^*).$$

$u_1'(y) > u_2'(y)$

- At a fair premium, the insurance buyer will always want to equalize marginal utilities of income across states.
- This implies equality of *incomes* across states if and only if the marginal utility of income is not state dependent.

$y_1^* > y_2^*$



Full insurance

- We can distinguish three ways to talk of “full insurance”:
  - choice of cover that equalizes marginal utilities of income across states
  - choice of cover that equalizes total utilities of income across states
  - choice of cover that equalizes income across states.
- When utility is state independent and the premium is fair, these three coincide: choice of cover equalizes incomes, marginal and total utilities.
- **Interesting implication:** An insurance contract that restricts cover to the loss actually incurred - actual loss on income from employment, actual medical costs, in the case of health insurance - is optimal only if marginal utility of income is state independent.