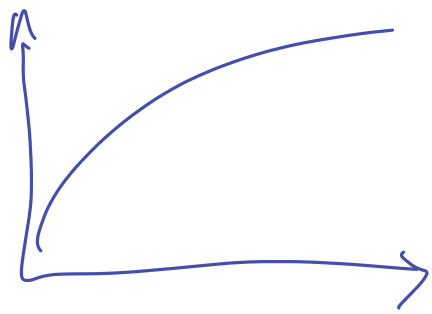
DAY 11

$u(y)$ $\qquad v(y) = \alpha + \beta\, u(y)$

$$v'(y) = \beta \cdot u'(y)$$

$$v''(y) = \beta \cdot u''(y)$$



Relative Risk Aversion

$$\boxed{R(y) = A(y) \cdot y}$$

$$= \frac{-u''(y)}{u'(y)} \cdot y$$

$\frac{\partial R(y)}{\partial y} > 0$ (IRRA)

$\frac{\partial R(y)}{} = 0$ (CRRA)

∂y

$$\frac{\partial R(y)}{\partial y} < 0 \qquad (\text{DRRA})$$

$$\frac{\partial R}{\partial y} = \boxed{A(y) + y \cdot \frac{\partial A(y)}{\partial y}}$$

$\oplus$ $\ominus$

Risk Preferences with $u(y) = \ln(y)$

$$A(y) = - \frac{u''(y)}{u'(y)}$$

$$u'(y) = \frac{1}{y} = y^{-1}$$

$$u''(y) = [y^{-1}]' = -y^{-2} = -\frac{1}{y^2}$$

$$A(y) = - \frac{-\frac{1}{y^2}}{1} = + \frac{1}{y}$$

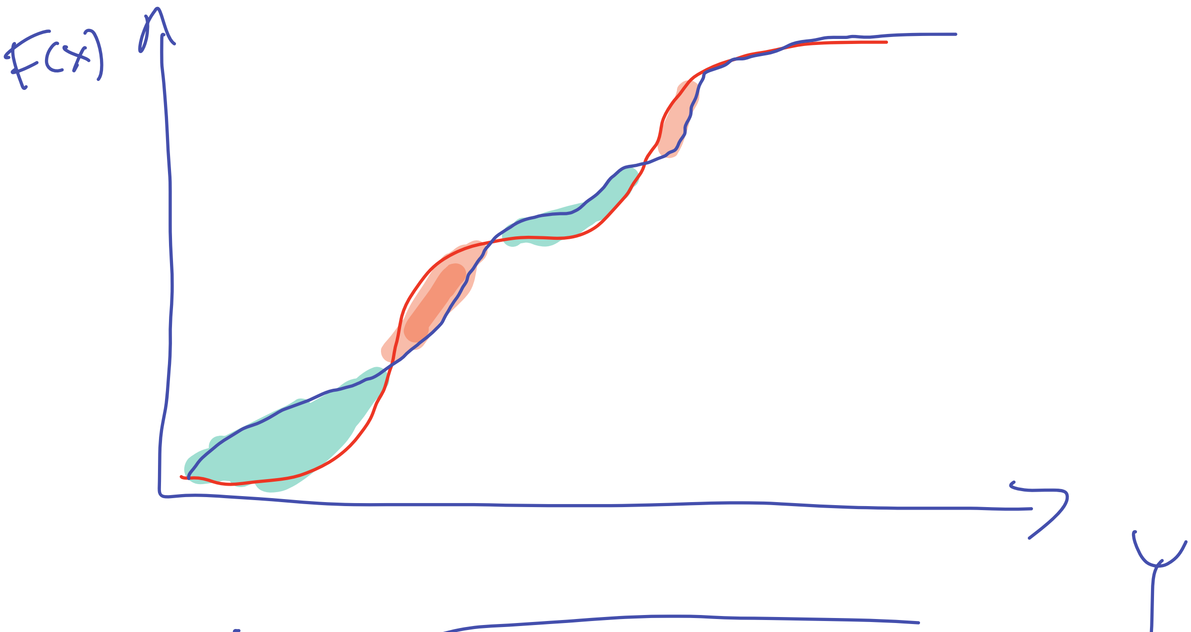
y

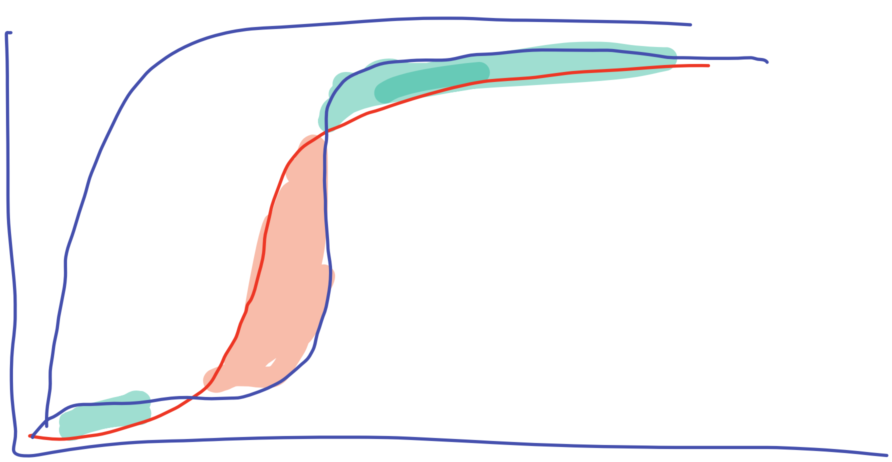
$$R(y) = A(y) \cdot y = \frac{1}{y} \cdot y = 1$$

$$\frac{\partial A(y)}{\partial y} = \left[y^{-1}\right]' = -\frac{1}{y^2} \qquad \text{DARA}$$

$$\frac{\partial R(y)}{\partial y} = (1)' = 0 \qquad \text{CRRA}$$

<u>Examples</u>





If $p = \pi \Rightarrow q^* = L$

FOC: $\cancel{p}(1-\cancel{\pi}) \cdot u'(\overbrace{Y - pq^*}) \overset{!}{=} (1-\cancel{p})\cancel{\pi} \cdot u'(Y - L + (1-p)q^*)$

$p = \pi$

$$u'(Y - pq^*) \overset{!}{=} u'(Y - L + (1-p)q^*) \quad \Big| (u')^{-1}$$

$$\cancel{Y} - \cancel{pq^*} = \cancel{Y} - L + \underbrace{\cancel{(1-p)q^*}}_{q^*}$$

$$\boxed{q^* = L}$$

If $p > \pi$

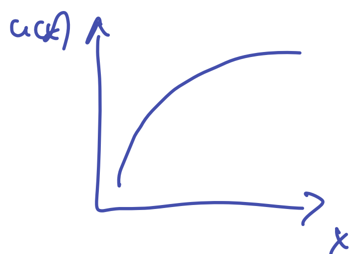
FOC $p(1-\pi) \cdot u'(Y - pq^*) = (1-p)\pi \cdot u'(Y - L + (1-p)q^*) \quad \Big|$ Divide

$$\frac{p(1-\pi)}{\pi(1-p)} = \frac{u'(Y - L + (1-p)q^*)}{u'(Y - pq^*)} > 1 \Leftrightarrow$$

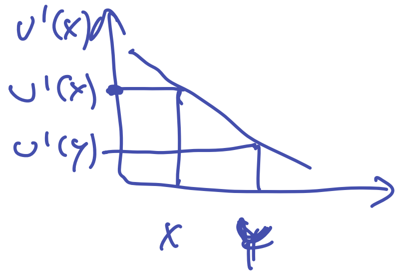
$\Rightarrow 1 \cdot > 1$

$$u'(Y - L + (1-p)q^*) > u'(Y - pq^*) \quad \Big| (u')^{-1}$$

$$\cancel{Y} - L + \cancel{(1-p)}q^* < \cancel{Y} - \cancel{pq^*}$$



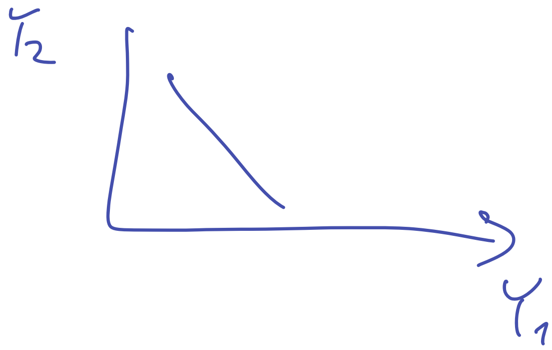
$$q^* < L$$



Slope of the BC

BC : $(1-p)\,Y_1 + p\,Y_2 = Y - pL$

↳ $Y_2(Y_1) =$



⟺ $pY_2 = Y - pL - (1-p)Y_1 \quad \Big| \cdot \frac{1}{p}$

$$\boxed{Y_2(Y_1) = \frac{Y - pL}{p} - \frac{(1-p)}{p} \cdot Y_1}$$

$$\boxed{\frac{dY_2}{dY_1} = \left(- \frac{1-p}{p}\right)}$$

Expected Value : $(1-\pi)\cdot Y_1 + \pi \cdot Y_2 = \bar{Y}$

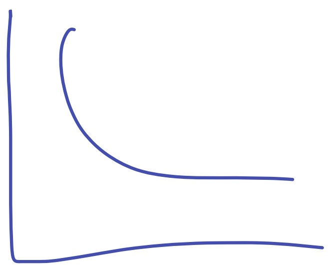
↳ Rearrange: $y_2(y_1) = \frac{\bar{Y}}{\pi} - \frac{1-\pi}{\pi} \cdot y_1$

$$\boxed{\hookrightarrow \frac{\partial y_2}{\partial y_1} = -\frac{1-\pi}{\pi}}$$

Indifference Curve

$$\boxed{EU = (1-\pi)\, u(y_1) + \pi \cdot u(y_2)}$$

$$MRS_{21} = \frac{\partial y_2}{\partial y_1}\Bigg|_{u = const} = -\frac{\partial EU / \partial y_1}{\partial EU / \partial y_2}$$



$$\boxed{= -\frac{(1-\pi)}{\pi} \cdot \frac{u'(y_1)}{u'(y_2)}} \quad MRS$$