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Economic Foundations and Applications of Risk, 2023

Exercises for Applications of Risk (Chapters 4–8)

1. **Portfolio choice: Example with logarithmic utility**
   Let an individual have a logarithmic utility function and an initial asset of W. This asset can be placed in a safe investment with return i, or in a risky investment with return x, where x either equals $\bar{x}$ (with probability p) or $\underline{x}$ (with probability $1-p$). Assume that $\underline{x} < i < \bar{x}$ and let the expected value of x equal $\mu > i$.

   (a) Write down the maximization problem of the individual and derive the first-order condition(s).

   (b) Determine the optimum amount a^* that the individual will invest in the risky project.

   (c) Derive the conditions under which a^* is larger than zero.

   (d) Show that a^* increases if initial wealth increases. What does this have to do with the measure of absolute risk aversion? How will the *proportion* of the asset invested into the risky project change if the asset increases? Why?

2. **Production decision with price uncertainty**
   Let the owner (and decision maker) of a firm have a logarithmic utility function. Assume her initial wealth to be $w_0 = 100$. The firm produces a commodity, a, with marginal cost of $c = 5$. The firm's maximum capacity is $\bar{a} = 20$. Let the fixed cost of production be $F = 2$. Under unfavorable market conditions (with probability 50%), the market price of the commodity is assumed to be $p_B = 3$, and otherwise $p_G = 9$.

   (a) Write down the expected-utility problem of the firm owner and calculate the optimal output a^* she would produce.

   (b) Argue verbally how a^* will react to a (moderate) increase in F. *(Note: This answer is not as trivial as it may seem at first sight.)* Verify your answer by calculating a^* for $F = 3$.

(c) Argue verbally how a^* will change if the price uncertainty changes to $p'_B = 4$ and $p'_G = 8$ (again, each with equal probability). Verify your answer by calculating a^* under these conditions. How much will the firm produce if the price is no longer uncertain but $p = 6$?

3. **The value of information**

Sebastian Vettel is starting in Monte Carlo. The weather outlook is bad, so that all of his rivals have wet tyres fitted. Just before the start, *Red Bull* gets the chance of obtaining a local weather update, which, however, would cost the team 2000 Euros. Suppose there are two states of the world, good weather (z_1) and bad weather (z_2), and two signals, s_1 (the sun comes through) and s_2 (it continues to be rainy). Let the matrix of the **common** probabilities ($Pr(s_i \cap z_j)$) be

	s_1	s_2
z_1	$0,2$	$0,1$
z_2	$0,2$	$0,5$

Vettel can choose between two actions: he can have fixed either dry tyres (a_1) or wet tyres (a_2). Depending on the state of the world he will receive the following payoffs:

	z_1	z_2
a_1	45000	0
a_1	20000	20000

(a) Determine the matrix of the ex-post probabilities (i.e. $Pr(z_j|s_i)$).

(b) Show that without a signal Vettel will drive with wet tyres.

(c) What is the signal worth? Will *Red Bull* pay the 2000 Euros?