Economic Foundations and Applications of Risk

0. Introduction

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Syllabus

- 0.1 What is risk?
- 0.2 Informational settings of decision problems
- 0.3 Notation
- 0.4 Some possible decision criteria

0.1 What is risk (in economics)?

Uncertainty = Absence of complete information

Different Outcomes, some worse than others

0.1 What is risk (in economics)?)

Risk

Uncertainty

Ambiguity

0.2 Informational settings of decision problems

1. Decision under certainty

a_1	$\rightarrow$	u_1
a_2	$\rightarrow$	u_2
.	$\rightarrow$	.
.	$\rightarrow$	.
.	$\rightarrow$	.

$$\max_{x_1, x_2} \quad u(x_1, x_2)$$

$$\hookrightarrow \quad a^* = \{x_1^*, x_2^*\}$$

- Agent's actions lead to a **certain** payoff. ($\rightarrow$ E.g. classic consumer or producer theory.)

2. Strategic interdependencies

Player b

Player a

Actions	b_1	b_2	...
a_1	(u_{11}, v_{11})	(u_{12}, v_{12})	...
a_2	(u_{21}, v_{21})	(u_{22}, v_{22})	...
.	.	.	...
.	.	.	...
.	.	.	...

- Agent's **payoff may** not solely **depend on** her own actions but also (in part) on **another agent's actions**. ($\rightarrow$ Game theory)

3. Decisions under uncertainty

Actions/States of the world	z_1	z_2	...
a_1	u_{11}	u_{12}	...
a_2	u_{21}	u_{22}	...
.	.	.	...
.	.	.	...
.	.	.	...

- Decisions under uncertainty can be modeled as games where the player that draws second (**nature**) does not act strategically but **picks her "action" according to a given probability distribution.**
- That probability distribution is usually assumed to be common knowledge amongst all the other players (inter alia to make sure their beliefs are consistent).
- Nature's actions are referred to as **"states of the world".**

Example for choices under uncertainty

Career choice and monthly salaries:

1. Bavarian civil servant: 5.000 EUR
2. Entrepreneur: If z_1: 20.000 EUR; if z_2: 1.000 EUR

Career choice is the choice between two lotteries: A secure one (L_1) and a risky one (L_2):

- Lottery $L_1 = (1, 0; 5.000, 0)$
- Lottery $L_2 = (p, 1 - p; 20.000, 1.000)$

$$L_i \quad (p_1, p; x_1, x_2)$$

4. Strategic interdependencies under uncertainty

- **Principal-agent problems**: The principal's payoff may not only depend on the agent's effort choice but may also be subject to the whims of fortune. ($\rightarrow$ Economics of information)

0.3 Notation

Example : 2 States of World → G, B

2 Actions / Lotteries CS, E

(a) **Choice variables (Actions): $\mathbf{A}$** $= \{a_1, \dots, a_n\}$

(b) **States of the world: $\mathbf{Z}$** $= \{z_1, \dots, z_m\}$

(c) **Probability vectors: $\mathbf{P}$** $= \{\mathbf{p}_1, \dots, \mathbf{p}_n\}$ with $\mathbf{p}_j = (p_{j1}, \dots, p_{jm})$ with the following properties: $0 \leq p_{ji} \leq 1 \; \forall i, j$; $\sum_{i=1}^{m} p_{ji} = 1 \; \forall i$

(d) **Payoff vectors $\mathbf{X}$** $= \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ with $\mathbf{x}_j = (x_{j1}, x_{j2}, \dots, x_{jm})$

(e) **Set of lotteries: $\mathbf{L}$** $= \{\mathbf{L}_1, \dots, \mathbf{L}_n\}$ $\mathbf{L}_j := (\mathbf{p}_j; \mathbf{x}_j)$

What we are after: $a_i^* \in \mathbf{A}$, that will optimize ...?

To solve this decision problem, we need a **preference ordering over lotteries!** In the next chapter we will look at some possible candidates.

$a^* \hat{=} L^*$

0.4 Some possible decision criteria

1. Expected value of a lottery

- **Criterion:** $\mathbf{L}_1 \succeq \mathbf{L}_2 \Leftrightarrow \mu_1 = \sum p_{1i}x_{1i} \geq \mu_2 = \sum p_{2i}x_{2i}$
- **Pro:** Simple. Under certain circumstances, evolution may favor expected-value maximizing individuals.
- **Con:** Risk is being ignored! Utility function: $U(L_j) = \mu_j$. Sensible approximation for e.g. risk-neutral firms, government. Why not for individuals?
- Consider the following example:

$$\begin{aligned} \mathbf{L}_1 &= \left(\tfrac{1}{2}, 0, \tfrac{1}{2}; 10.000, 5.000, 10\right) \\ \mathbf{L}_2 &= (0, 1, 0; 10.000, 5.000, 10) \\ &\Rightarrow \mu_1 = 5.005 > \mu_2 = 5.000 \end{aligned}$$

$L_1 \xrightarrow{\frac{1}{2}} 10{,}000$

$L_1 \xrightarrow{\frac{1}{2}} 10$

$L_2 \xrightarrow{1} 5000$

2. The maximin criterion

- **Criterion:** $\mathbf{L}_1 \succeq \mathbf{L}_2 \Leftrightarrow min_i[x_{1i}|p_{1i} > 0] \geq min_i[x_{2i}|p_{2i} > 0]$
- **Pro:** Takes account of losses/unfavorable outcomes.
- **Con:** Losses may be over-emphasized (unlikely as they may be).
- Consider the following example:

$$\begin{aligned} \mathbf{L}_1 &= \left(\tfrac{99}{100}, \tfrac{1}{100}; 10.000, 0\right) \\ \mathbf{L}_2 &= (0, 1; 10.000, 5.000) \\ &\Rightarrow \mathbf{L}_2 \succeq \mathbf{L}_1 \end{aligned}$$

99% 10,000

L_1

1% 0

$L_2 \rightarrow 5000$

, E(L)

3. The $\mu - \sigma$ criterion

Risk

- **Basic idea:** Agents like a big payout (μ), but they dislike risk (σ).
- **Example utility:** $U(\mathbf{L}_j) = f(\mu_j, \sigma_j) = \mu_j - k\sigma_j$ with $\mu_j = \sum_i p_{ji}x_{ji}$ and $\sigma_j^2 = \sum_i p_{ji}(x_{ji} - \mu_j)^2$ where k is a measure of an agent's risk-aversion.
- **Pro:** Rather intuitive. Easy to handle. This criterion is commonly used in capital markets theory ($\rightarrow$ CAPM).
- **Con:** Further distribution moments (e.g. curtosis) are ignored.

4. Expected utility

- **Historic origin:** St. Petersburg Paradox
  - Swiss mathematician **Daniel Bernoulli (1738)** came up with a solution on how to value the chance to participate in the following game, the so-called St. Petersburg Paradox:
  - Toss a coin time and again for as long as it is showing tails
  - As soon as it is shows heads, the game is over and the gambler, who bought into the bet, will get a pay-out of $2^{n\cancel{+1}}$, where n is the number of times the coin had previously been showing tails.
  - **Expected value** of payout:

$$\mu = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot \frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 2^3 + \ldots = 1 + 1 + 1 + \ldots = +\infty$$

  - **Willingness to pay** for participating in the game?

- **Bernoulli's proposal:** Decisions should not depend on (expected) payouts but on (expected) **utility** from payouts.
  - "*. . . the incremental utility from a gain is inversely related to a player's wealth. . .*"
- $\rightarrow$ **Concave** utility function (E.g. $u(x) = \ln(x)$)
- **Expected utility** of payout:

$$U(\mathbf{L}) = \frac{1}{2} \cdot \ln(2) + \frac{1}{4} \cdot \ln(4) + \frac{1}{8} \cdot \ln(8) + \ldots \ll +\infty$$

- Looks alright, but **can this criterion be generalized?**
- We will answer this question in the next chapter.