$$E(MR) = E(MC)$$

Economic Foundations and Applications of Risk

Part B. Applications

Chapter 6: Firms under Uncertainty

Till Stowasser

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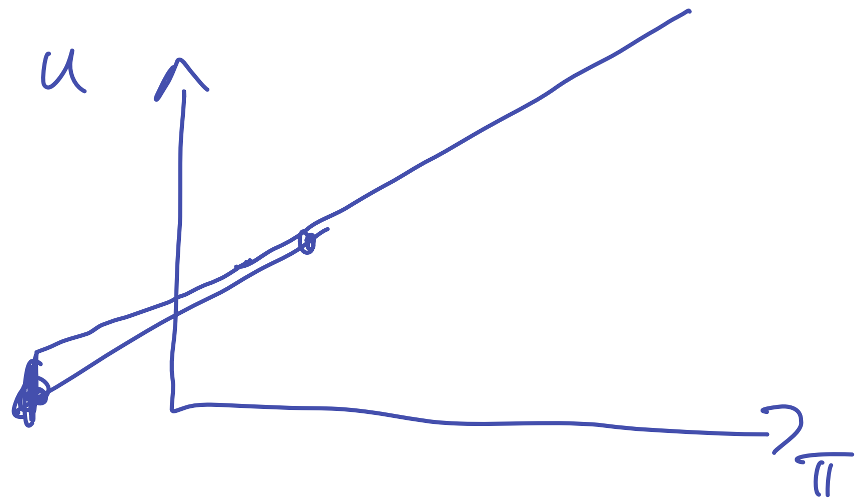
Syllabus



- 6.1 Introduction
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- 6.3 Uncertainty in production decisions
- 6.4 Uncertainty in investments decisions

6.1 Introduction

- The **presence of risk influences** not only the decisions of individuals . . .
- . . . but also the **decisions of firms.**
- We start with a few remarks on whether firms should be realistically viewed as **risk-averse or risk-neutral** (6.2).
- We continue by assessing the impact on **production decisions,** if there is uncertainty about key market or technology parameters (6.3).
- We close by addressing the option value of delaying decisions when there is uncertainty about the profitability of **investment** projects (6.4).

6.2 Risk attitudes of firms

- For reasons of risk spreading and risk sharing, **firms are often modeled as risk-neutral agents.**
- However, there are **various reasons** why firms may be considered as **risk-averse decision makers:**
- **Agency problems** within the firm
  - Contract theory: Incentive pay is used to overcome moral hazard in the owner-manager relationship (i.e. the manager's pay is a function of firm performance).
  - Consequence: Instead of maximizing the risk-neutral owners' expected return, risk-averse managers maximize their own utility.
- **Quasi-concave payout functions**
  - Bankruptcy cost: The risk of bankruptcy leads to non-linearities in the payout schedule. (Loss of firm-specific human capital or the customer base.)
  - Convex tax schedules: Increasing marginal tax rates make higher gross profit less valuable.

6.3 Uncertainty in production decisions

Uncertainty over model parameters

- We will augment the firm's standard production-decision problem with uncertainty over key model parameters.
- There could be uncertainty over **market conditions** (e.g. input factor prices, or selling prices).
- There could be uncertainty over **technology** (e.g. the cost function, or the production function).
- We will study the case of uncertainty over the selling price.

Selling-price uncertainty

- **Setup:** Let w_0 be the firm's initial wealth, a the amount of output produced, $c(a)$ the cost of producing output a, and $\tilde{p}$ the (uncertain) selling price.
- Then, owner's final wealth, $\tilde{w}_f$, is given by $\tilde{w}_f = w_0 + \tilde{p}a - c(a)$.
- The **decision maker will maximize** $\mathrm{E}[u(\tilde{w}_f)]$. $u(w_f(a))$

$$E[X \cdot Y]$$

$$\textbf{FOC:}\quad \frac{d\mathrm{E}[u(\tilde{w}_f)]}{da} = \mathrm{E}\Big[\underbrace{u'(w_0 + \tilde{p}a - c(a))}_{}\cdot\underbrace{(\tilde{p} - c'(a))}_{}\Big] \stackrel{!}{=} 0$$

- Recall that

$$X \mathrel{\hat{=}} \frac{\partial u'}{\partial w_f}. \qquad Y = \frac{\partial w_f}{\partial a}$$

$$\cancel{\mathrm{Cov}(x,y) = \mathrm{E}[xy] - \mathrm{E}[x]\mathrm{E}[y]} \iff \mathrm{E}[xy] = \mathrm{Cov}(x,y) + \mathrm{E}[x]\mathrm{E}[y]$$

$$E[X+c] = E[X]+c$$

- Hence, the FOC is equivalent to:

$$\underbrace{\mathrm{Cov}(u'(\cdot), (\tilde{p} - c'(a)))}_{= \mathrm{Cov}(X,Y)} + \underbrace{\mathrm{E}[u'(\cdot)]}_{E(X)} \cdot \underbrace{E[\tilde{p} - c'(a)]}_{E(Y)} \stackrel{!}{=} 0$$

$E[\tilde{p}] - c'(a)$

- Since $c'(a)$ is not a random variable, the FOC reduces to

FOC

$$\mathrm{Cov}(u'(\cdot), \tilde{p}) + \mathrm{E}[u'(\cdot)](\mathrm{E}[\tilde{p}] - c'(a)) \stackrel{!}{=} 0$$

$$\iff \boxed{\mathrm{E}[\tilde{p}] = c'(a) - \frac{\mathrm{Cov}(u'(\cdot), \tilde{p})}{\mathrm{E}[u'(\cdot)]}}$$

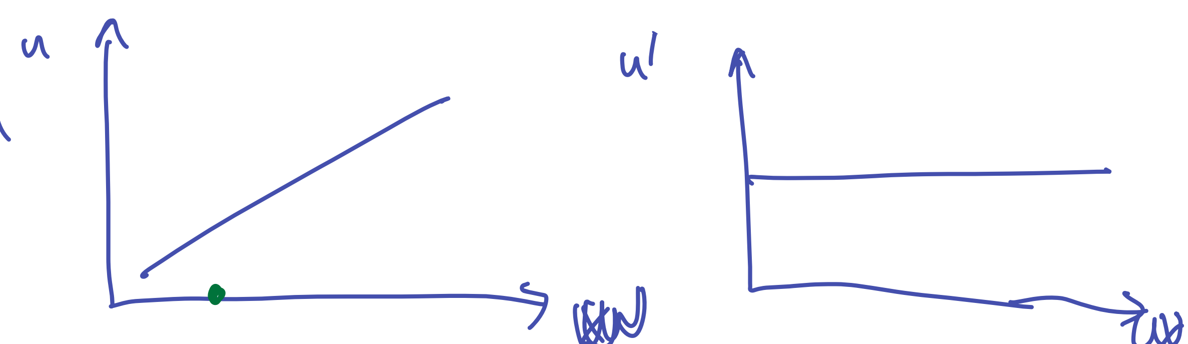
$= 0$ if risk-neutral

$\ominus$ if risk-averse

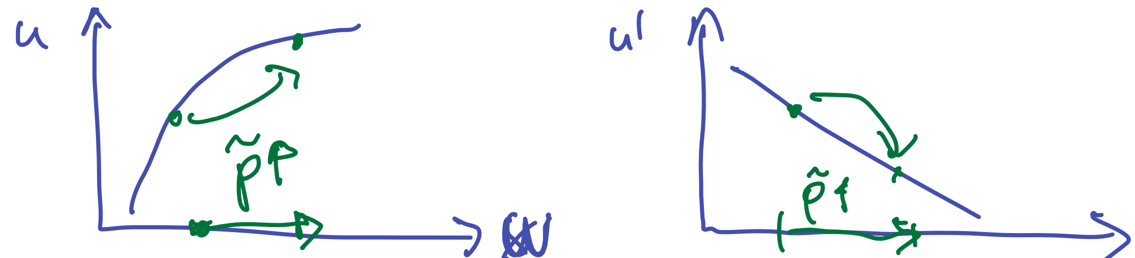
- **If the firm is risk-neutral,** this optimality condition reduces to $\mathrm{E}[\tilde{p}] = c'(a)$, since risk neutrality implies linear utility:

$$u' = \text{const.} \Rightarrow \mathrm{Cov}(u'(\cdot), \tilde{p}) = 0.$$

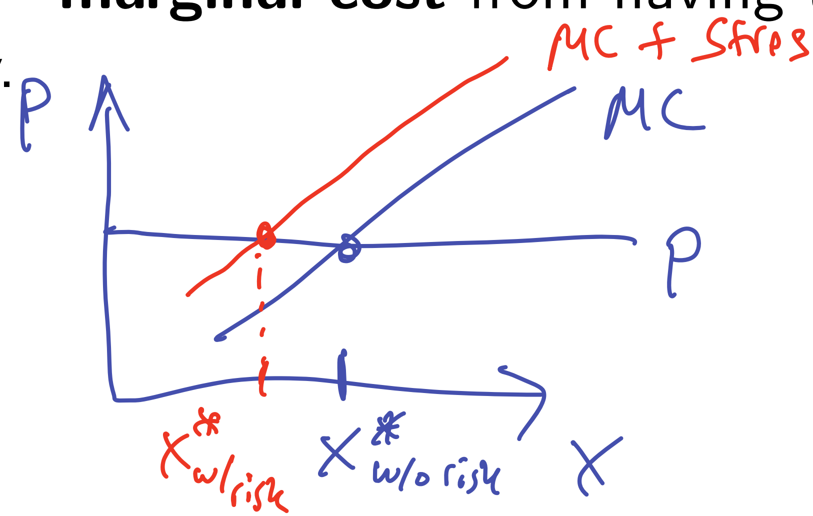
Risk-neutral



Risk averse



- **If the firm is risk-averse,** the FOC becomes $\mathrm{E}[\tilde{p}] > c'(a)$, inducing **reduced production** when compared to a situation without risk:
  - Ceteris paribus, as $\tilde{p}$ decreases, so does final wealth $\tilde{w}_f$, which implies increasing $u'(\tilde{w}_f)$.
  - Hence, $\mathrm{sgn}\{\mathrm{Cov}(u'(\cdot), \tilde{p})\} < 0$, implying $-\frac{\mathrm{Cov}(u'(\cdot),\tilde{p})}{\mathrm{E}[u'(\cdot)]} > 0$.
- Intuitively, one may think of $-\frac{\mathrm{Cov}(u'(\cdot),\tilde{p})}{\mathrm{E}[u'(\cdot)]}$ as some kind of additional **"psychological" marginal cost** from having to produce under uncertainty.



6.4 Uncertainty in investment decisions

Uncertainty over the investment return

- We will augment the firm's standard investment-decision problem with uncertainty over the return of the investment project.
- Let there be one (irreversible) investment project, which triggers one-time costs of I.
- Its return $\tilde{\pi}(t)$, $t = 1, \ldots$ is assumed to be uncertain.
- **Standard theory** suggests the following **decision rule:** Invest iff NPV ≥ 0,

$$\iff E[\sum_{t=1}^{\infty} \delta^t \tilde{\pi}(t)] - I \geq 0,$$

where $\delta \equiv \frac{1}{1+r}$ is the discount factor and r the real interest rate.

Real-option theory of investment

- However, the so-called real-option theory (ROT) of investment **comes to a different conclusion.**
- If there is an **option to delay** the investment project, one can let time work in ones favor and reduce the uncertainty.
- This is closely related to chapter 8 which addresses **the value of information.**
- The following example illustrates the intuition of ROT.

ROT example

- Setup:
  - $I = 1600$ EUR
  - $r = 0.1$
  - In $t = 0$, the project will certainly yield a payoff of 200 EUR.
  - In $t \in [1;\infty]$, the project will with equal probability yield a constant annual payoff of either 100 EUR or 300 EUR.
- According to **standard theory,** the **project should be undertaken:**
  - $\text{NPV} = \boxed{E[\sum_{t=0}^{\infty} \delta^t \tilde{\pi}(t)] - I} = 200 + \sum_{t=1}^{\infty} \delta^t \frac{300+100}{2} - I$
    $= \sum_{t=0}^{\infty} \frac{200}{(1{,}1)^t} - 1600 = \boxed{\frac{200}{1-\frac{1}{1{,}1}}} - 1600 = 600 > 0$
- According to **ROT,** however, it may be wiser to **postpone the investment decision** by one period.
- The firm can make itself better off by **buying an option to invest** rather than committing to investing right away

$\frac{100}{1-\frac{1}{1.1}}$

Changed Decision

- **What would the firm do if it waited** one period ($t = 1$)?
  - If $\pi(t = 1) = 100 \Rightarrow \text{NPV}(t = 1) = 1100 - 1600 < 0$. In this case, the project should be refrained from.
  - If $\pi(t = 1) = 300 \Rightarrow \text{NPV}(t = 1) = 3300 - 1600 > 0$. In this case, the project should be commenced. $= 1700$

    $\frac{300}{1-\frac{1}{1.1}}$ ← Not changed Decision

- **What is the NPV** at $t = 0$ **of the following strategy:** Wait for one period and invest iff $\pi(t = 1) = 300$?

$$\text{NPV}(t = 0) = \underbrace{\frac{1}{2}}_{\text{Prob}(\pi=100)} \cdot 0 + \underbrace{\frac{1}{2}}_{\text{Prob}(\pi=300)} \cdot \frac{\text{NPV}(t = 1)}{1,1} = \frac{1}{2} \cdot \frac{1700}{1,1} = 773$$

- Recall that the project where the firm had to make its decision in $t = 0$ had a $\text{NPV}(t = 0) = 600$.
- Hence, the **value of the flexibility option** F is 173 EUR.
  - 173 is the maximum amount the firm would be willing to buy for the option to make the investment

- This flexibility introduces **three effects:**
  - The firm will lose 200 EUR in $t = 0$.
  - The firm will gain $I(1-\delta)$, because investment costs are due one period later. Hence, $I \uparrow \Rightarrow F \uparrow$
  - The firm will get an annual return of 300 EUR instead of $\frac{300+100}{2}$ EUR (in expectation), because it no longer has to bear that uncertainty.
- It is **not always optimal to wait**
  - In our example $\pi_0 = 200$ and $\pi_1 = 1,5\pi_0$ or $\pi_1 = 0,5\pi_0$ with equal probability.
  - If $\pi_0 < 97$ it is not optimal to invest at all (even the good case will make a loss)
  - If $\pi_0 > 249$ it is optimal to invest in any case (even the bad case will make a profit)
  - Only if $97 < \pi_0 < 249$, it is optimal to wait and invest only if the good case arises

Final remarks on ROT

Here: Perfect signal

- In this context, waiting is **equivalent to buying a signal.**
  - The firm buys the signal by foregoing a certain profit of 200 EUR in $t = 0$.
  - We will see in chapter 8, that a rational decision maker will never seek costly information unless there is a chance that the information may actually change what she is going to do.
- ROT is used in **various contexts:**
  - Labor markets (job offers, job search)
  - Oil Reserves
  - Product Development (e.g. electric cars)
  - R&D
  - Law changes
  - Marriage
  - Suicide
  - ...