

Economic Foundations and Applications of Risk

0. Introduction

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0.2 Informational settings of decision problems

1. Decision under certainty

a_1	\rightarrow	u_1
a_2	\rightarrow	u_2
.	\rightarrow	.
.	\rightarrow	.
.	\rightarrow	.

- Agent's actions lead to a **certain** payoff. (\rightarrow E.g. classic consumer or producer theory.)

Example for choices under uncertainty

Career choice and monthly salaries:

- 1 Bavarian civil servant: 5.000 EUR
- 2 Entrepreneur: If z_1 : 20.000 EUR; if z_2 : 1.000 EUR

Career choice is the choice between two lotteries: A secure one (L_1) and a risky one (L_2):

- Lottery $L_1 = (1, 0; 5.000, 0)$
- Lottery $L_2 = (p, 1 - p; 20.000, 1.000)$

4. Strategic interdependencies under uncertainty

- **Principal-agent problems:** The principal's payoff may not only depend on the agent's effort choice but may also be subject to the whims of fortune. (→ Economics of information)

0.3 Notation

- (a) **Choice variables (Actions):** $\mathbf{A} = \{a_1, \dots, a_n\}$
- (b) **States of the world:** $\mathbf{Z} = \{z_1, \dots, z_m\}$
- (c) **Probability vectors:** $\mathbf{P} = \{\mathbf{p}_1, \dots, \mathbf{p}_n\}$ with $\mathbf{p}_j = (p_{j1}, \dots, p_{jm})$
with the following properties: $0 \leq p_{ji} \leq 1 \forall i, j; \sum_{i=1}^m p_{ji} = 1 \forall j$
- (d) **Payoff vectors** $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ with $\mathbf{x}_j = (x_{j1}, x_{j2}, \dots, x_{jm})$
- (e) **Set of lotteries:** $\mathbf{L} = \{\mathbf{L}_1, \dots, \mathbf{L}_n\}$ $\mathbf{L}_j := (\mathbf{p}_j; \mathbf{x}_j)$

What we are after: $a_i^* \in \mathbf{A}$, that will optimize ...?

To solve this decision problem, we need a **preference ordering over lotteries!** In the next chapter we will look at some possible candidates.

0.4 Some possible decision criteria

1. Expected value of a lottery

- **Criterion:** $L_1 \succeq L_2 \Leftrightarrow \mu_1 = \sum p_{1i}x_{1i} \geq \mu_2 = \sum p_{2i}x_{2i}$
- **Pro:** Simple. Under certain circumstances, evolution may favor expected-value maximizing individuals.
- **Con:** Risk is being ignored! Utility function: $U(L_j) = \mu_j$. Sensible approximation for e.g. risk-neutral firms, government. Why not for individuals?
- Consider the following example:

$$L_1 = \left(\frac{1}{2}, 0, \frac{1}{2}; 10.000, 5.000, 10\right)$$

$$L_2 = (0, 1, 0; 10.000, 5.000, 10)$$

$$\Rightarrow \mu_1 = 5.005 > \mu_2 = 5.000$$

2. The maximin criterion

- **Criterion:** $\mathbf{L}_1 \succeq \mathbf{L}_2 \Leftrightarrow \min_i [x_{1i} | p_{1i} > 0] \geq \min_i [x_{2i} | p_{2i} > 0]$
- **Pro:** Takes account of losses/unfavorable outcomes.
- **Con:** Losses may be over-emphasized (unlikely as they may be).
- Consider the following example:

$$\mathbf{L}_1 = \left(\frac{99}{100}, \frac{1}{100}; 10.000, 0 \right)$$

$$\mathbf{L}_2 = (0, 1; 10.000, 5.000)$$

$$\Rightarrow \mathbf{L}_2 \succeq \mathbf{L}_1$$

3. The $\mu - \sigma$ criterion

- **Basic idea:** Agents like a big payout (μ), but they dislike risk (σ).
- **Example utility:** $U(\mathbf{L}_j) = f(\mu_j, \sigma_j) = \mu_j - k\sigma_j$ with $\mu_j = \sum_i p_{ji}x_{ji}$ and $\sigma_j^2 = \sum_i p_{ji}(x_{ji} - \mu_j)^2$ where k is a measure of an agent's risk-aversion.
- **Pro:** Rather intuitive. Easy to handle. This criterion is commonly used in capital markets theory (\rightarrow CAPM).
- **Con:** Further distribution moments (e.g. curtosis) are ignored.

4. Expected utility

■ **Historic origin:** St. Petersburg Paradox

- Swiss mathematician **Daniel Bernoulli (1738)** came up with a solution on how to value the chance to participate in the following game, the so-called St. Petersburg Paradox:
- Toss a coin time and again for as long as it is showing tails
- As soon as it shows heads, the game is over and the gambler, who bought into the bet, will get a pay-out of 2^{n+1} , where n is the number of times the coin had previously been showing tails.
- **Expected value** of payout:

$$\mu = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot \frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 2^3 + \dots = 1 + 1 + 1 + \dots = +\infty$$

- **Willingness to pay** for participating in the game?

- **Bernoulli's proposal:** Decisions should not depend on (expected) payouts but on (expected) **utility** from payouts.
 - "... *the incremental utility from a gain is inversely related to a player's wealth...*"
- → **Concave** utility function (E.g. $u(x) = \ln(x)$)
- **Expected utility** of payout:

$$U(\mathbf{L}) = \frac{1}{2} \cdot \ln(2) + \frac{1}{4} \cdot \ln(4) + \frac{1}{8} \cdot \ln(8) + \dots \ll +\infty$$

- Looks alright, but **can this criterion be generalized?**
- We will answer this question in the next chapter.