Problem 1: Preferences over lotteries

8

a) [~~7~~ Points]

- $P = (1 \,;\, 200)$
- $Q = (0.5, 0.3, 0.1, 0.1 \,;\, 0, 200, 400, 1000)$

→ $E(P) = 200$

→ $E(Q) = 200$

→ But $P \succ^{SOSD} Q$

OR

A = B, but due to risk aversion, A carries more weight than B.

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b) + c) [~~8~~ Points] 4 + 8

- $P = (1 \,;\, 200)$
- $Q' = (0.5, 0.3, 0.1, 0.1 \,;\, 0, 200, 450, 1000)$

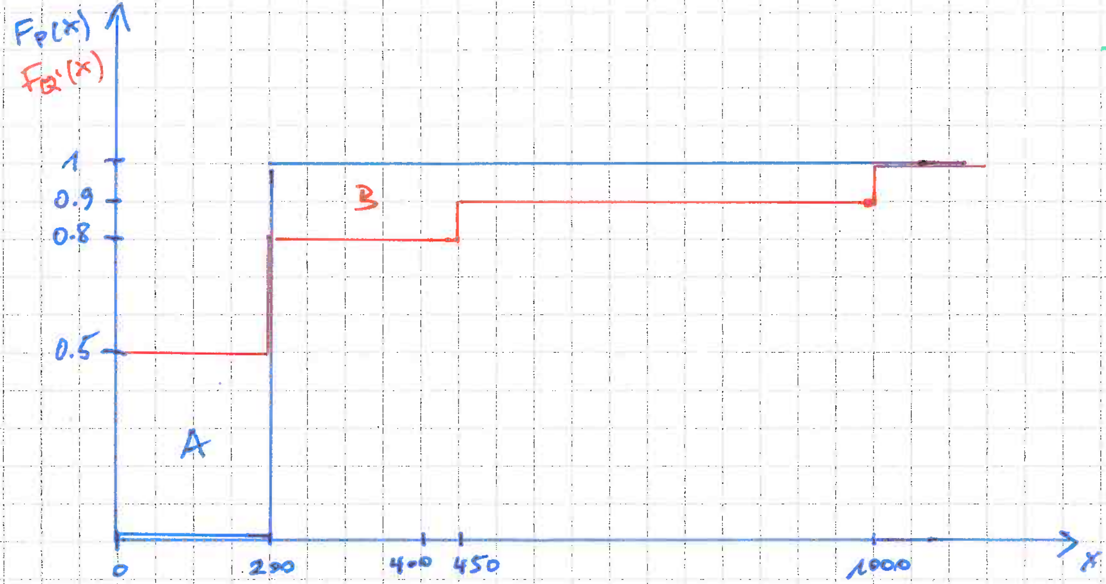
→ $E(P) = 200$

→ $E(Q') = 205$

→ So Q' has higher expected payout but carries more risk

↳ SOSD no longer works to sort P and Q'

b) 4P



Not necessary to draw this

A < B ⇒ No SOSD !

· However, we know the following!

① $G = (0.5, 0.5 ; 450, 1000)$ with $Eu(G) = u(650)$

② $Q' \sim Q'' = (0.5, 0.3, 0.2 ; 0, 200, G)$

⇒ Combine ① and ② :

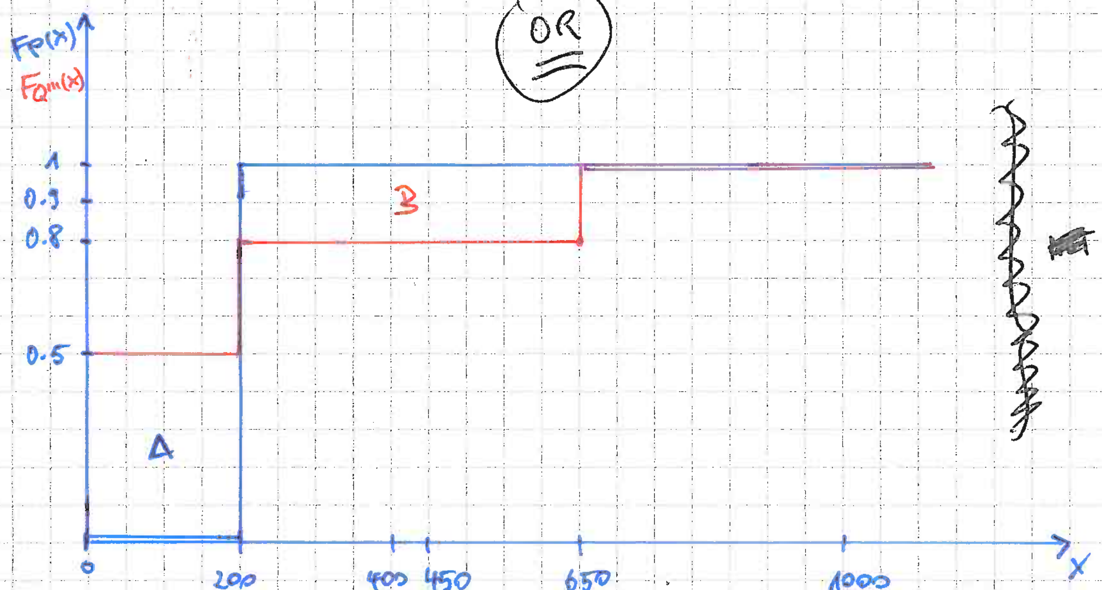
$Q' \sim Q'' \sim Q''' = (0.5, 0.3, 0.2 ; 0, 200, 650)$

· $E(Q''') = 190 < E(P) = 200$

↳ SOSD works again : $P \succ^{SOSD} Q''' \sim Q' \Rightarrow P \succ^{SOSD} Q'$

c) 8P

OR



A > B ⇒ SOSD ✓

Problem 2: Flexibility option and signal system

a) [~~10~~ 8 points] 8P

→ Payout matrix

	z_1	z_2
a_1	1,000	10,000
a_2	7,000	7,000

→ $Pr(s_i \cap z_j)$

	s_1	s_2	
z_1	0.2	0.1	$Pr(z_1) = 0.3$
z_2	0.2	0.5	$Pr(z_2) = 0.7$
	$Pr(s_1) = 0.4$	$Pr(s_2) = 0.6$	

→ $Pr(s_i \mid z_j)$

	s_1	s_2	
z_1	2/3	1/3	$\Sigma = 1$
z_2	2/7	5/7	$\Sigma = 1$

Not necessary to show

→ $Pr(z_j \mid s_i) = \boxed{\dfrac{Pr(s_i \cap z_j)}{Pr(s_i)}}$

	s_1	s_2
z_1	1/2	1/6
z_2	1/2	5/6
	$\Sigma = 1$	$\Sigma = 1$

8P

b) [~~9~~ 8 Points] 8P

- $E[X \mid a_1] = 0.3 \cdot 1{,}000 + 0.7 \cdot 10{,}000 = 7{,}300$ €
- $E[X \mid a_2] = 7{,}000$ €

↳ Choose a_1 : Make A favorite

c) ~~8~~ points] 10P

- Cost of signal: 500€

- If s_1: bad performance on day 3
- $E[x \mid a_1 \wedge s_1] = \frac{1}{2} \cdot 1{,}000 + \frac{1}{2} \cdot 10.000 = 5{,}500€$
- $E[x \mid a_2 \wedge s_1] = 7{,}000€$

↳ Choose a_2: Make B favorite (Strategy change!)

- If s_2: good performance on day 3
- $E[x \mid a_1 \wedge s_2] = 1/6 \cdot 1{,}000 + 5/6 \cdot 10{,}000 = 8{,}500€$
- $E[x \mid a_2 \wedge s_2] = 7{,}000€$

↳ Choose a_1: Make A favorite

Alternative:

$$V = 0.4\Big[\underbrace{\left(\tfrac{1}{2}\cdot 7000 + \tfrac{1}{2}\cdot 7000\right)}_{=7000} - \underbrace{\left(\tfrac{1}{2}\,1000 + \tfrac{1}{2}\cdot 10{,}000\right)}_{=5{,}500}\Big] + \underbrace{0.6\Big[\left(\tfrac{1}{6}\cdot 1000 + \tfrac{5}{6}\cdot 1000\right) - \left(\tfrac{1}{6}\,1000 + \tfrac{5}{6}\,10{,}000\right)\Big]}_{=0}$$

$$= 0.4(7000 - 5500) = 600$$

- Value of waiting:
- $E[x \mid \text{Waiting}] - E[x \mid \text{Not Waiting}]$ ⟺ OR in here w/ negative sign!

$$[\underbrace{0.4}_{Pr(s_1)} \cdot 7{,}000 + \underbrace{0.6}_{Pr(s_2)} \cdot 8{,}500] - \underbrace{7{,}300}_{\text{see b)}} = 600€ > 500€$$

$Pr(s_1)$ → Make B favorite; $Pr(s_2)$ → Make A favorite; see b) → Make A favorite

↳ Team manager will wait!

- The maximum cost of waiting that still makes the manager wait is 600.

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d) ~~8~~ Points] 8P

- Compare Payouts with and without waiting

→ Payout of waiting

$E[X \mid \text{Waiting}] = 0.4 \cdot 7{,}000 + 0.6 \cdot 8.500 - 500 = 7.400€$

→ Payout of no waiting and 450 if a_2

- $E[x \mid a_1] = 7.300€$ (see b))
- $E[x \mid a_2] = 7{,}000 + 450 = 7.450€$ ← Don't wait and make B favorite

e) [6 points]

- Mistake in Problem description!
  - It should have said: How does your answer in d) change ...
- Since this was my mistake, every student gets full marks for e): 6/6!
- Here is the correct solution, though:

→ Solution in d) was to choose the safe option of selecting B (No risk)

⇒ If team manager now becomes risk-averse, he/she will be even more inclined to avoid risk than before. So the team-manager's decision will NOT change, compared to d).