Economic Foundations and Applications of Risk (2023)

Exam

October 20th, 2023

Instructions

- You are supposed to **answer all questions**.
- You have **60 minutes for a total of 60 points.**
- **Show how you derived your answers** (either by calculation or by verbal reasoning).
- Your answers can be in **German or English** (or any combination that I am able to understand).
- **You may use** a (non-programmable) **calculator**.
- Questions marked with an **asterisk** (*) are **meant to be a bit trickier** than the rest, so don't despair if you find them hard to crack.
- **Good luck!!!**

1. **Being a lab rat [10 points]**

   You are in LMU's laboratory for experimental economics, 'MELESSA', playing the following 'split the dollar' game: You are given 20 EUR. You can offer your opponent either 10 EUR or 5 EUR. If your opponent accepts the offer, you get to keep the rest of the money. If your opponent rejects the offer, both you and your opponent will get nothing. With probability p, your opponent is a pragmatist who will accept any offer. With probability $1-p$, your opponent is a proud person who will only accept an equal share of the 20 EUR.

   (a) Explain how your decision problem can be modeled as the choice of a vNM-utility maximizer between two lotteries. [5 points]

   (b) Let your preferences be described by $u(x) = \sqrt{x}$. For what critical probability p are you exactly indifferent between offering 10 EUR or 5 EUR. [5 points]

2. **Going to a rock concert [12 points]**

   The Rolling Stones have decided to end their World tour in Munich's 'Atomic Cafe'. There are 10000 fans interested in attending but as the 'Atomic Cafe' is quite small, the number of tickets available is limited to 500.

   The 'Atomic Cafe' has decided to sell admission via a lottery. They print 10000 lottery tickets, work out two proposals and ask you – knowing a lot about the economic foundations and applications of risk – which of the two proposals provides maximum benefit for the audience. The two proposals are:

   A: Charge nothing for the lottery ticket. The 500 lottery winners get a concert ticket in exchange for $P = 100$ EUR. (This right is non-transferable.)

   B: Charge $p = 5$ EUR for a lottery ticket and give the 500 winners free admission to the concert. (This right is non-transferable.)

   Assume that the lottery is fair, (i.e. everybody has a 5% chance of winning), that all 10000 possible visitors have identical (positive) risk aversion, identical initial income ($\bar{w}$), an identical willingness to pay for the concert ticket that exceeds 100 EUR and for the lottery ticket that exceeds 5 EUR. Suppose that the ticket creates utility for an individual with a monetary equivalent of $W > 100$ EUR.

   Which proposal is favored by a risk-averse, expected-utility-maximizing lottery participant? Explain your answer verbally and/or with a suitable diagram. [12 points]

3. **Different risk preferences [38 points]**

Consider the utility functions of the following three individuals, where w stands for wealth.

- $u_A(w) = w^\alpha$, with $\alpha = 1$
- $u_B(w) = w^\beta$, with $\beta = 2$
- $u_C(w) = w^\gamma$, with $\gamma = \frac{1}{2}$

(a) What can you say about the risk preferences of these three individuals? Verify your statements by calculating the marginal rates of substitution for each individual and by drawing their indifference curves into a two-states-of-the-world diagram. [15 points]

(b) Suppose individual B has an initial wealth of 1 monetary unit (MU) and owns a lottery ticket that pays out 3 MUs with a probability of 20% and nothing with a probability of 80%. What is the minimum amount of money you would have to offer to individual B for her to sell the ticket to you? How high is individual B's risk premium? [7 points]

*(c) Draw the situation described in (b) into a two-states-of-the-world diagram. Make sure to depict individual B's initial risk allocation, her indifference curve that goes through that initial allocation, her certainty equivalent, and her risk premium. *(Hint: You can use the indifference curve of a risk-neutral individual as a benchmark to depict the risk premium in this diagram.)* [8 points]

Now consider the utility functions of the following two individuals, where w still stands for wealth.

- $u_D(w) = \ln(w)$
- $u_E(w) = \sqrt{w}$

(d) Assume that the initial wealth of both individuals equals $w = 100$ and that they both have the following investment opportunity: $L = (\frac{1}{2}, \frac{1}{2}; +5, -5)$. Is individual D or individual E more risk averse towards this lottery? Answer this question by using a measure of risk aversion that fits this situation. [8 points]