Risk Exam 2023 – Solution

1 Lab Rat

(a) [5 Points]

- The vNM utility maximiser has the choice between two lotteries
  - Lottery 1: Safe choice: Offer 10 EUR → Opponent will accept → Secure payout of 10 EUR

    ↳ $L_1 = (1;\ 10)$

  - Lottery 2: Risky choice: Offer 5 EUR
    - p → Opponent will accept → Payout of 15 EUR
    - $(1-p)$ → Opponent will refuse → Payout of 0 EUR

    ↳ $L_2 = (p, (1-p);\ 15, 0)$

(b) [5 Points]

- Decisionmaker will be indifferent between both lotteries if

$$E[u(L_1)] = E[u(L_2)] \iff$$

$$\sqrt{10} = p \cdot \sqrt{15} + \underbrace{p\sqrt{0}}_{=0} \iff$$

$$\sqrt{10} = p\sqrt{15} \iff$$

$$p = \frac{\sqrt{10}}{\sqrt{15}} \approx 0.816$$

↳ If the probability of the opponent being a pragmatist equals 81.6%, I am indifferent between offering 10 EUR or 15 EUR

2 Rock Concert

Setup:
- $N = 10{,}000$ (No. of fans who want a ticket) = (No of lottery tickets printed)
- $n = 500$ (No. of fans who get to go $\hat{=}$ capacity)

- 2 Proposals

  A
  - Lottery tickets are free
  - Winning tickets let you see the concert for $P = 100$ EUR

  B
  - Lottery tickets cost 5 EUR $= p$
  - Winning tickets let you see the concert for no additional cost

- Assumptions
  - Fair lottery (Chance of winning = 5%)
  - All identical risk aversion
  - All identical initial income, $\bar{w}$ } $\hat{=}$ Same risk preferences
  - All identical w.t.p. > 100 EUR for concert, > 5 EUR for ticket } $\hat{=}$ Excess supply & everyone will buy a lottery ticket

Which proposal is favoured by risk-averse person? [12 Points]

- Let's start by comparing the average payout of both proposals

~~E[$\bar{w}$(LA)]~~

$$\rightarrow E(L_A) = 0.95 \cdot \bar{w} + 0.05\left(\bar{w} - 100 + W\right)$$

$$= \underbrace{0.95 \cdot \bar{w} + 0.05\,\bar{w}}_{\bar{w}} - 5 + 0.05 \cdot W$$

$$\bar{w} - 5 + 0.05\,W$$

$$\rightarrow E(L_B) = 0.95 \cdot \left(\bar{w} - 5\right) + 0.05\left(\bar{w} - 5 + W\right)$$

$$= 0.95\,\bar{w} - 0.95 \cdot 5 + 0.05\,\bar{w} - 0.05 \cdot 5 + 0.05\,W$$

$$\bar{w} - 5 + 0.05\,W$$

$\Longrightarrow$ Both options give the same expected payout

[4P]

Let's compare cumulative distributions to see if one option stochastically dominates the other

Option A : Best Case : $\bar{\omega} - \overset{=100}{P} + W$ Recall: $(W > P)$

Worst Case : $\bar{\omega}$

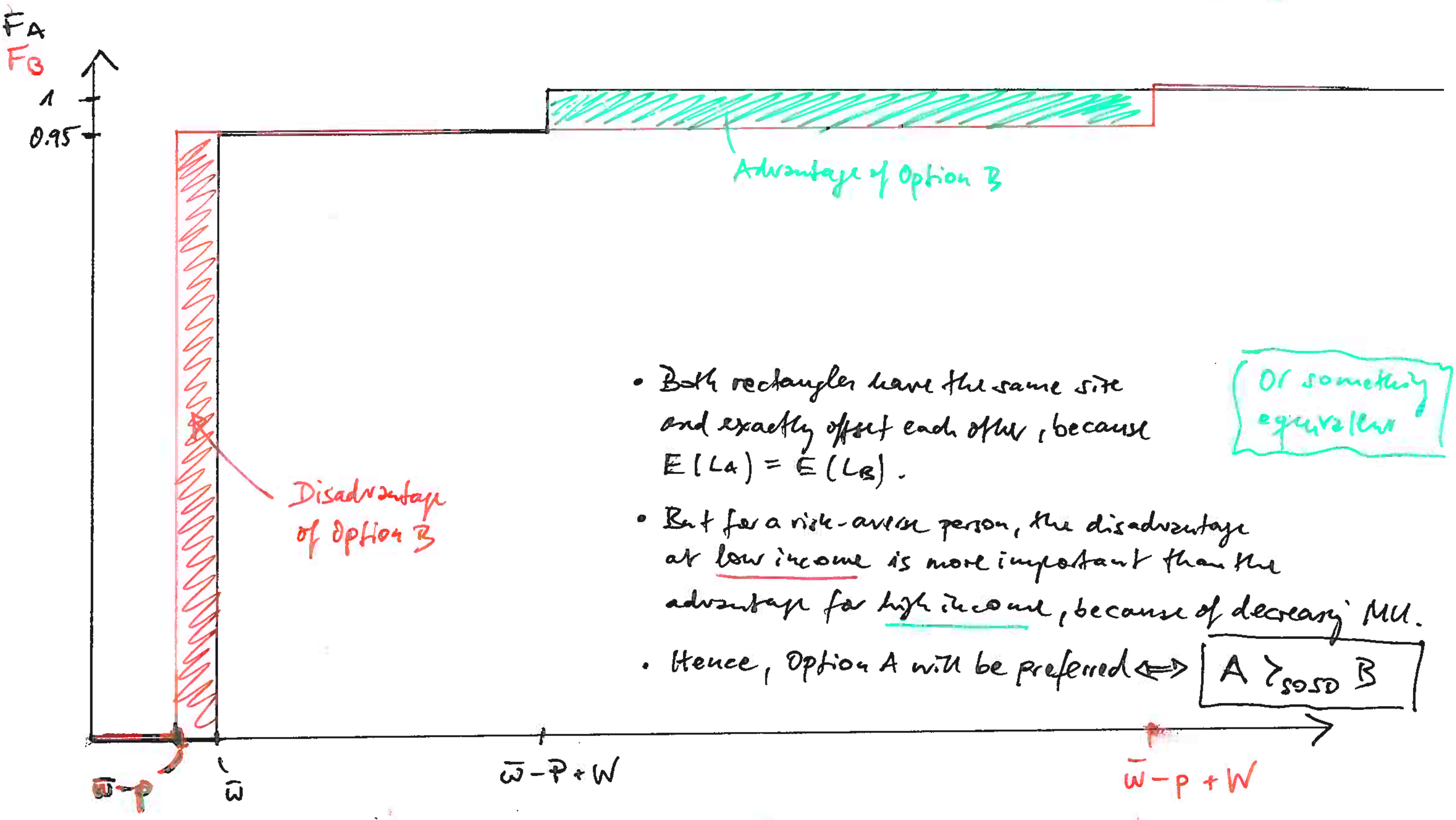
Option B : Best Case : $\bar{\omega} - p^{=5} + W$ $\Rightarrow$ Best Case got better

Worst Case : $\bar{\omega} - p_{=5}$ $\Rightarrow$ Best Case got worse

So Option B is a MPS of Option A

↳ Risk-averse people will prefer Option A

Optional (Either/Or) [8P]



- Both rectangles have the same size and exactly offset each other, because $E(L_A) = E(L_B)$.
- But for a risk-averse person, the disadvantage at low income is more important than the advantage for high income, because of decreasing MU.
- Hence, Option A will be preferred $\Leftrightarrow$ $\boxed{A \succsim_{SOSD} B}$

Or something equivalent

3 Different risk preferences

Setup:

- $u_A(w) = w^{\alpha} \quad (\alpha = 1) \Rightarrow \underline{u_A(w) = w}$ (Linear utility)
- $u_B(w) = w^{\beta} \quad (\beta = 2) \Rightarrow \underline{u_B(w) = w^2}$ (Quadratic utility)
- $u_C(w) = w^{\gamma} \quad (\gamma = \frac{1}{2}) \Rightarrow \underline{u_C(w) = \sqrt{w}}$ (Squareroot utility)

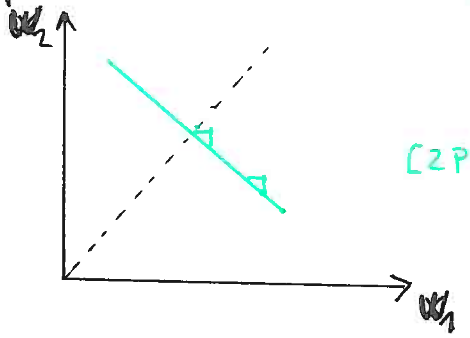
a) Risk preferences [15 Points]

$$\boxed{EU = p \cdot u(w_1) + (1-p)\, u(w_2)}$$

- Individual A has a linear utility function and is <u>risk-neutral</u> [1P]

$$\hookrightarrow MRS_{21} = \frac{dw_2}{dw_1} = -\frac{MU_1}{MU_2} = -\frac{1}{1}\frac{p}{1-p} = -1 \cdot \frac{p}{1-p}$$

← Slope of IC is constant [2P]



[2P]

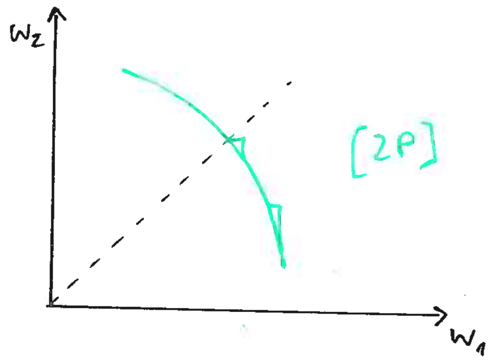
- Individual B has a quadratic utility function and is <u>risk-loving</u> [1P]

$$\hookrightarrow MRS_{21} = \frac{dw_2}{dw_1} = -\frac{MU_1}{MU_2} = -\frac{2w_1}{2w_2}\frac{p}{(1-p)} = -\frac{w_1\uparrow}{w_2}\cdot\frac{p}{1-p}\uparrow$$

Slope of IC becomes steeper in w_1

↳ Concave IC

[2P]



[2P]

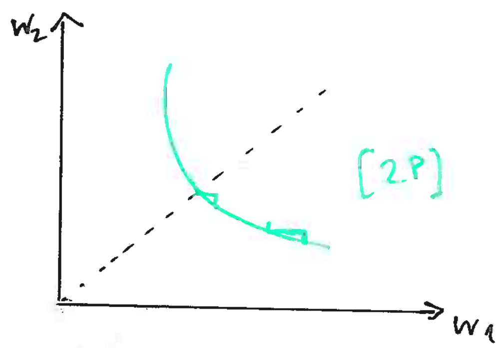
- Individual C has a squareroot utility function and is <u>risk-averse</u> [1P]

$$\hookrightarrow MRS_{21} = \frac{dw_2}{dw_1} = -\frac{MU_1}{MU_2} = -\frac{\frac{1}{2}w_1^{-\frac{1}{2}}}{\frac{1}{2}w_2^{-\frac{1}{2}}}\frac{p}{1-p} = -\left(\frac{w_2}{w_1\uparrow}\right)^{\frac{1}{2}} \cdot \frac{p}{1-p}\downarrow$$

Slope of IC becomes flatter in w_1

↳ Convex IC

[2P]



[2P]

b) Certainty Equivalent for Individual B [7P]

- Lottery:

$$\bullet \begin{cases} \xrightarrow{0.2} & w_g = 4 \\ \xrightarrow[0.8]{} & w_b = 1 \end{cases}$$

- Expected Utility from Lottery:

$$E[V_B] = 0.2 \cdot 4^2 + 0.8 \cdot 1^2$$

$$= 3.2 + 0.8 = 4$$

- Certainty Equivalent:
  - How much do I have to pay to B, such that she is indifferent between that payment, $\hat{w}$, and the lottery

$$E[V_B] = 4 \stackrel{!}{=} \hat{w}^2 \iff$$

$$2 = \hat{w} \leftarrow \text{Certainty Equivalent} \quad [4P]$$

- Risk Premium:

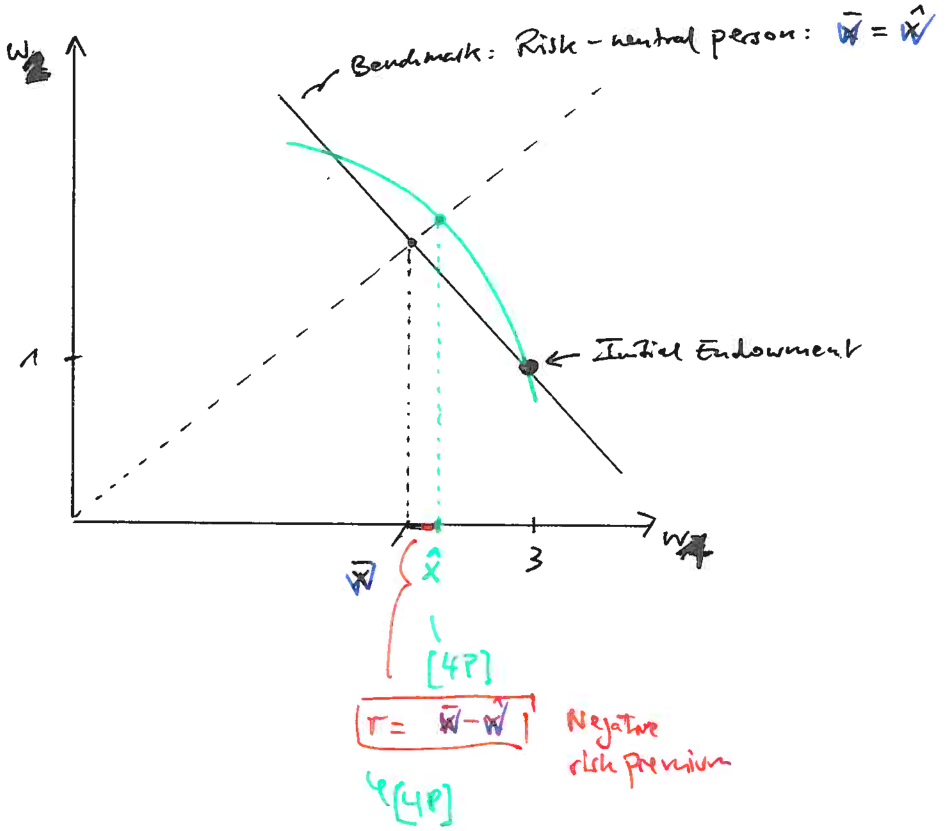
$$\boxed{r = \bar{w} - \hat{w}} \qquad \bar{w} = E(w) = 0.2 \cdot 4 + 0.8 \cdot 1 = 1.6 = \bar{w}$$

$$\hookrightarrow r = 1.6 - 2 = -0.4 \leftarrow \text{Negative risk premium}$$

↳ You would have to pay a risk lover to give up risk!

[3P]

c) 2-States of the World Diagram [8 Points]



d) Comparison of risk aversion [8 Points]

<u>Setup</u>:
- $u_D(w) = \ln(w)$
- $u_E(w) = \sqrt{w}$
- $w_0 = 100$
- <u>Lottery</u>: $L = \left(\frac{1}{2}, \frac{1}{2}; +5, -5\right)$

2 Options to answer who is more risk averse

Either/or

Option 1: Absolute Risk Aversion Measure

<u>Individual D</u>: [3P]

$$u'_D = \frac{1}{w}, \quad u''_D = -\frac{1}{w^2} \;\Rightarrow\; A_D(w) = -\frac{u''_D}{u'_D} = \frac{1}{w}$$

$$\Rightarrow A_D(100) = \frac{1}{100}$$

<u>Individual E</u>: [4P]

$$u'_E = \frac{1}{2} w^{-\frac{1}{2}}, \quad u''_E = -\frac{1}{4} w^{-\frac{3}{2}} \;\Rightarrow\; A_E(w) = -\frac{u''_E}{u'_E} = -\frac{-\frac{1}{4} w^{-\frac{3}{2}}}{\frac{1}{2} w^{-\frac{1}{2}}}$$

$$= \frac{1}{2w}$$

$$A_D(100) = \frac{1}{200}$$

D is more risk averse than E [1P]

Option 2: Risk premia

$\boxed{r = \bar{x} - \hat{x}}$ $\qquad \bar{x} = \frac{1}{2} 95 + \frac{1}{2} \cdot 105 = 100$

<u>Individual D</u>:

CE: $\hat{x}_D$: $u(\hat{x}_D) \equiv E[u(x)]$
$= \frac{1}{2} \cdot \ln(95) + \frac{1}{2} \ln(105)$
$u(\hat{x}_D) = 4.604$
$\ln(\hat{x}_D) = 4.604$
$\hat{x} = e^{4.604} \approx 99.883$

$r_A \approx 100 - 99.883 \approx 0.117$

<u>Individual E</u>:

CE: $\hat{x}_E$: $\sqrt{\hat{x}} = \frac{1}{2}\sqrt{95} + \frac{1}{2}\sqrt{105}$
$\sqrt{\hat{x}} = 9.997$
$\hat{x} = (9.997)^2 = 99.937$

$r_B \approx 100 - 99.937 \approx 0.063$

D is more risk averse than E