Economic Foundations and Applications of Risk

0. Introduction

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LMU, 2026

Planned Schedule

10 - 13 Part 1

13 - 14 Break

14 - 17 Part 2

Syllabus

- 0.1 Background reading
- 0.2 What is risk?
- 0.3 Informational settings of decision problems
- 0.4 Notation
- 0.5 Some possible decision criteria

0.1 Background Reading

- Banerjee, A. V. [1992] *A Simple Model of Herd Behavior*, The Quarterly Journal of Economics, pp. 797–817
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- Rothschild, M. and J. E. Stiglitz [1970] *Increasing Risk I: A Definition*, J. of Economic Theory , pp. 225–243
- Savage, L. J. [1954] *The Foundations of Statistics*, Wiley
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- Von Neumann, J. and O. Morgenstern [1953] *Theory of Games and Economic Behavior*, Princeton University Press

0.2 What is risk (in economics)?

having to make (economic) decisions under uncertainty
that may lead to undesirable outcomes

Knight (1921)

Kahneman/Tversky (1737)

Risk	Probabilities are known
Uncertainty	Probabilities are unknown but beliefs can be formed
Ambiguity	Probabilities are unknown and no rational beliefs

0.3 Informational settings of decision problems

1. Decision under certainty

Actions Utility

a_1	$\rightarrow$	u_1
a_2	$\rightarrow$	u_2
.	$\rightarrow$	.
.	$\rightarrow$	.
.	$\rightarrow$	.

Deterministic

E.g.: $\max_{x_1, x_2} u(x_1, x_2)$

- Agent's actions lead to a **certain** payoff. ($\rightarrow$ E.g. classic consumer or producer theory.)

$a^* \equiv \{x_1^*, x_2^*\}$

Game Theory

2. Strategic interdependencies

Agent b

Actions	b_1	b_2	...
a_1	(u_{11}, v_{11})	(u_{12}, v_{12})	...
a_2	(u_{21}, v_{21})	(u_{22}, v_{22})	...
.	.	.	...
.	.	.	...
.	.	.	...

Agent a

- Agent's **payoff may** not solely **depend on** her own actions but also (in part) on **another agent's actions**. ($\rightarrow$ Game theory)

3. Decisions under uncertainty

Nature

Actions/States of the world	z_1	z_2	...
a_1	u_{11}	u_{12}	...
a_2	u_{21}	u_{22}	...
.	.	.	...
.	.	.	...
.	.	.	...

Agent a

- Decisions under uncertainty can be modeled as games where the player that draws second (**nature**) does not act strategically but **picks her "action" according to a given probability distribution.**
- That probability distribution is usually assumed to be common knowledge amongst all the other players (inter alia to make sure their beliefs are consistent).
- Nature's actions are referred to as **"states of the world".**

Example for choices under uncertainty

Career choice and monthly salaries:

1. Bavarian civil servant: 5.000 EUR
2. Entrepreneur: If z_1: 20.000 EUR; if z_2: 1.000 EUR

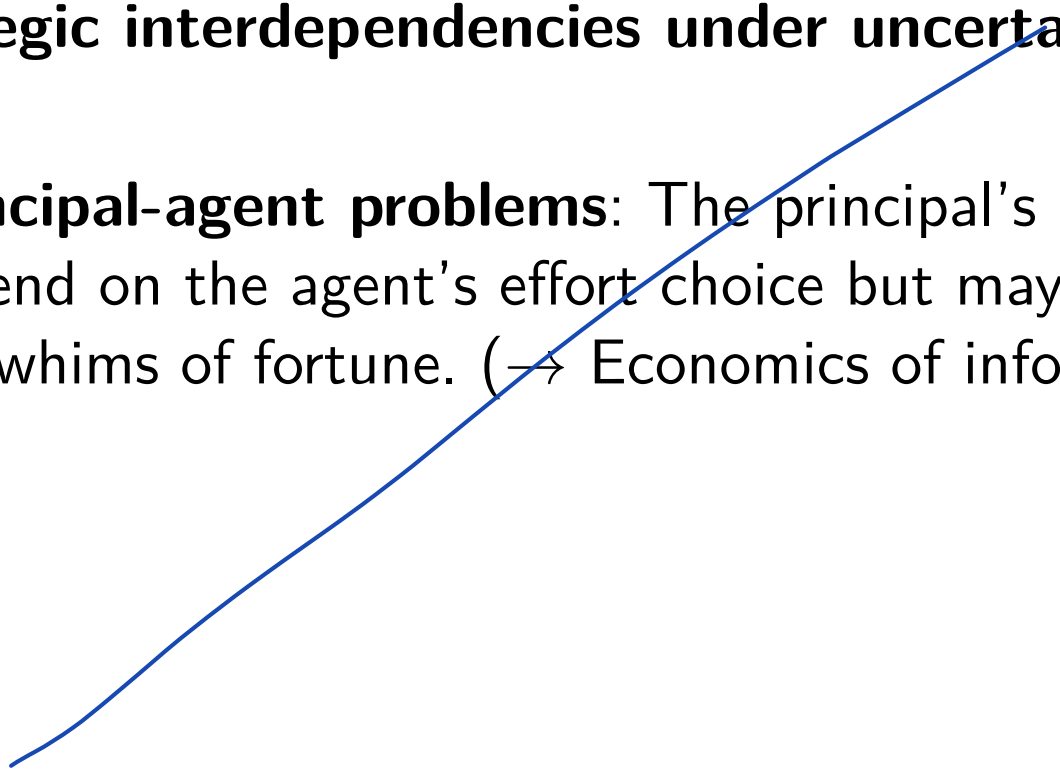
Career choice is the choice between two lotteries: A secure one (L_1) and a risky one (L_2):

- Lottery $L_1 = (1, 0; 5.000, 0)$ $= (1; 5000)$
- Lottery $L_2 = (p, 1-p; 20.000, 1.000)$

$$L \equiv (p_1, p_2; x_1, x_2)$$

4. Strategic interdependencies under uncertainty

- **Principal-agent problems**: The principal's payoff may not only depend on the agent's effort choice but may also be subject to the whims of fortune. ($\rightarrow$ Economics of information)



0.4 Notation

$A = \begin{cases} a_1 = BCC \\ a_2 = E \end{cases}$ $Z = \begin{cases} z_1 = G \\ z_2 = B \end{cases}$ $P = \begin{cases} p_1 = (1, 0) \\ p_2 = (p_1(1-p)) \end{cases}$ $X = \begin{cases} x_{1G} = 5000 \\ x_{1B} = 0 \\ x_{2G} = 20000 \\ x_{2B} = 1000 \end{cases}$

(a) **Choice variables (Actions): $\mathbf{A} = \{a_1, ..., a_n\}$**

(b) **States of the world: $\mathbf{Z} = \{z_1, ..., z_m\}$**

(c) **Probability vectors: $\mathbf{P} = \{\mathbf{p}_1, ..., \mathbf{p}_n\}$** with $\mathbf{p}_j = (p_{j1}, ..., p_{jm})$ with the following properties: $0 \leq p_{ji} \leq 1 \; \forall i, j$; $\sum_{i=1}^{m} p_{ji} = 1 \; \forall i$

(d) **Payoff vectors $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n\}$** with $\mathbf{x}_j = (x_{j1}, x_{j2}, ..., x_{jm})$

(e) **Set of lotteries: $\mathbf{L} = \{\mathbf{L}_1, ..., \mathbf{L}_n\}$ $\mathbf{L}_j := (\mathbf{p}_j; \mathbf{x}_j)$**

What we are after: $a_i^* \in \mathbf{A}$, that will optimize ...?

To solve this decision problem, we need a **preference ordering over lotteries!** In the next chapter we will look at some possible candidates.

$L_1 = (1; 5000)$ $L_2 = (p_1(1-p); 20{,}000; 1000)$

0.5 Some possible decision criteria

1. Expected value of a lottery

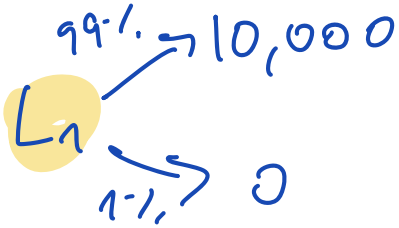
- **Criterion:** $\mathbf{L}_1 \succeq \mathbf{L}_2 \Leftrightarrow \mu_1 = \sum p_{1i} x_{1i} \geq \mu_2 = \sum p_{2i} x_{2i}$
- **Pro:** Simple. Under certain circumstances, evolution may favor expected-value maximizing individuals.
- **Con:** Risk is being ignored! Utility function: $U(L_j) = \mu_j$. Sensible approximation for e.g. risk-neutral firms, government. Why not for individuals?
- Consider the following example:

Mean

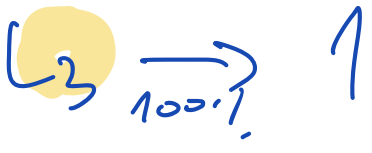
$$\begin{aligned} \mathbf{L}_1 &= (\tfrac{1}{2}, 0, \tfrac{1}{2}; 10.000, 5.000, 10) \\ \mathbf{L}_2 &= (0, 1, 0; 10.000, 5.000, 10) \\ &\Rightarrow \quad \mu_1 = 5.005 > \mu_2 = 5.000 \end{aligned}$$

2. The maximin criterion

- **Criterion:** $\mathbf{L}_1 \succeq \mathbf{L}_2 \Leftrightarrow min_i[x_{1i}|p_{1i} > 0] \geq min_i[x_{2i}|p_{2i} > 0]$
- **Pro:** Takes account of losses/unfavorable outcomes.
- **Con:** Losses may be over-emphasized (unlikely as they may be).
- Consider the following example:



$$\begin{aligned} \mathbf{L}_1 &= (\tfrac{99}{100}, \tfrac{1}{100}; 10.000, 0) \\ \mathbf{L}_2 &= (0, 1; 10.000, 5.000) \\ &\Rightarrow \mathbf{L}_2 \succeq \mathbf{L}_1 \end{aligned}$$



$L_3 (0,1 ; 10000,1)$

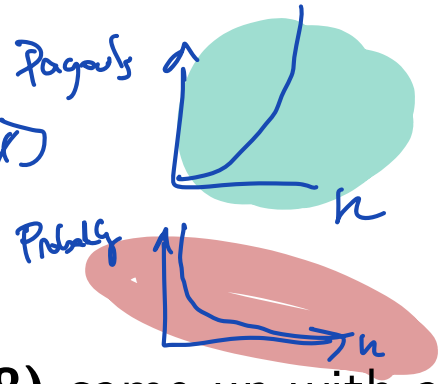
Mean of Lottery

StdDev of Lottery

3. The $\mu - \sigma$ criterion

- **Basic idea:** Agents like a big payout (μ), but they dislike risk (σ).
- **Example utility:** $U(\mathbf{L}_j) = f(\mu_j, \sigma_j) = \mu_j - k\sigma_j$ with $\mu_j = \sum_i p_{ji} x_{ji}$ and $\sigma_j^2 = \sum_i p_{ji}(x_{ji} - \mu_j)^2$ where k is a measure of an agent's risk-aversion.
- **Pro:** Rather intuitive. Easy to handle. This criterion is commonly used in capital markets theory ($\rightarrow$ CAPM).
- **Con:** Further distribution moments (e.g. curtosis) are ignored.

4. Expected utility

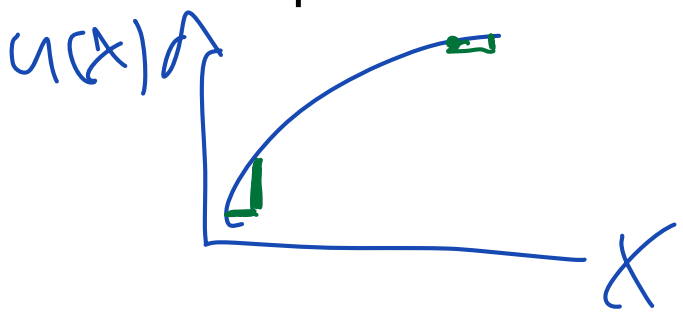


- **Historic origin:** St. Petersburg Paradox
  - Swiss mathematician **Daniel Bernoulli (1738)** came up with a solution on how to value the chance to participate in the following game, the so-called St. Petersburg Paradox:
  - Toss a coin time and again for as long as it is showing tails
  - As soon as it is shows heads, the game is over and the gambler, who bought into the bet, will get a pay-out of 2^n, where n is the number of times the coin has been showing tails.
  - **Expected value** of payout:

$$\mu = \tfrac{1}{2} \cdot 2 + \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot 2^2 + \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot 2^3 + \ldots = 1 + 1 + 1 + \ldots = +\infty$$

  - **Willingness to pay** for participating in the game?

- **Bernoulli's proposal:** Decisions should not depend on (expected) payouts but on (expected) **utility** from payouts.
  - "... *the incremental utility from a gain is inversely related to a player's wealth*..."
- $\rightarrow$ **Concave** utility function (E.g. $u(x) = \ln(x)$)
- **Expected utility** of payout:

$$U(\mathbf{L}) = \tfrac{1}{2} \cdot \ln(2) + \tfrac{1}{4} \cdot \ln(4) + \tfrac{1}{8} \cdot \ln(8) + \ldots \ll +\infty$$

- Looks alright, but **can this criterion be generalized?**
- We will answer this question in the next chapter.