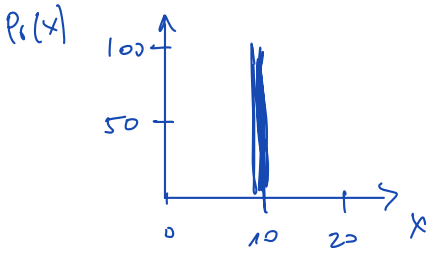
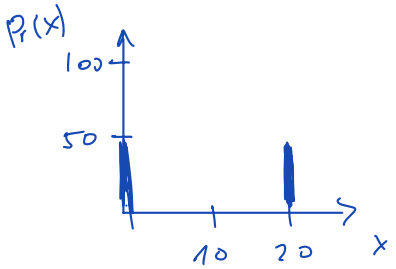
DAY 2

Motivation Chapter 3

So far, we compared Secure payments with risky lotteries

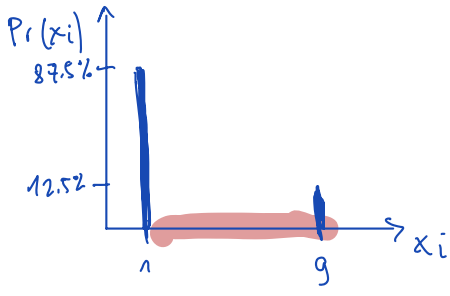
e.g.: $L_1 = (1; 10)$ vs. $L_2 = (\frac{1}{2}, \frac{1}{2}; 0, 20)$

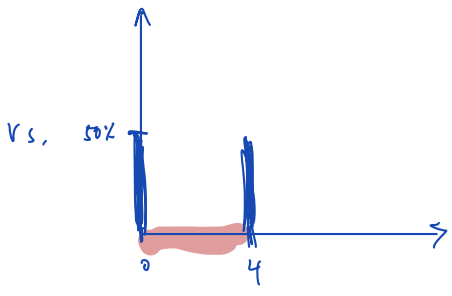




But in practice, we often want to compare two (or more) risky lotteries

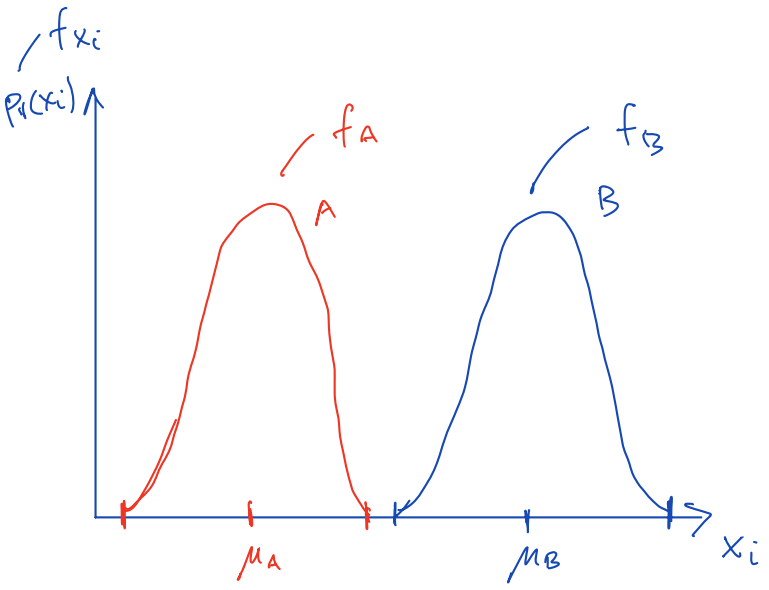
e.g.: $L_1 = (\frac{7}{8}, \frac{1}{8}; 1, 9)$ vs. $L_2 = (\frac{1}{2}, \frac{1}{2}; 0, 4)$





Some examples : Which lottery would you choose ?

Strict domination

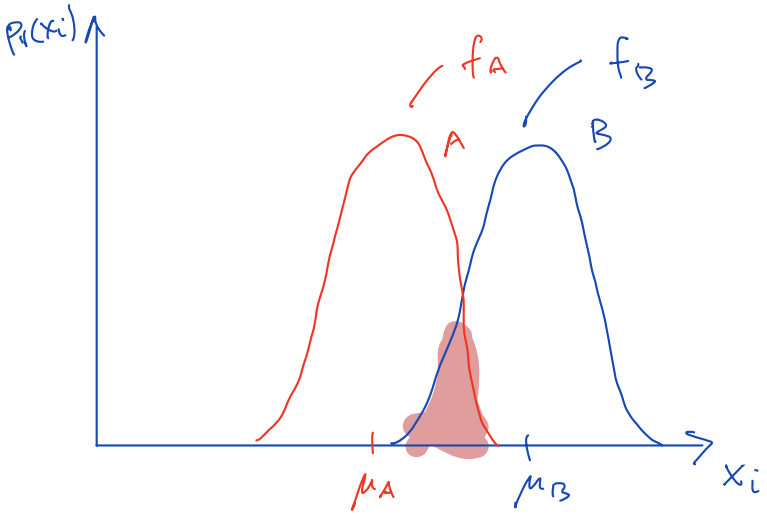


Risk averse → B

Risk neutral → B

Risk loving → B

⇒

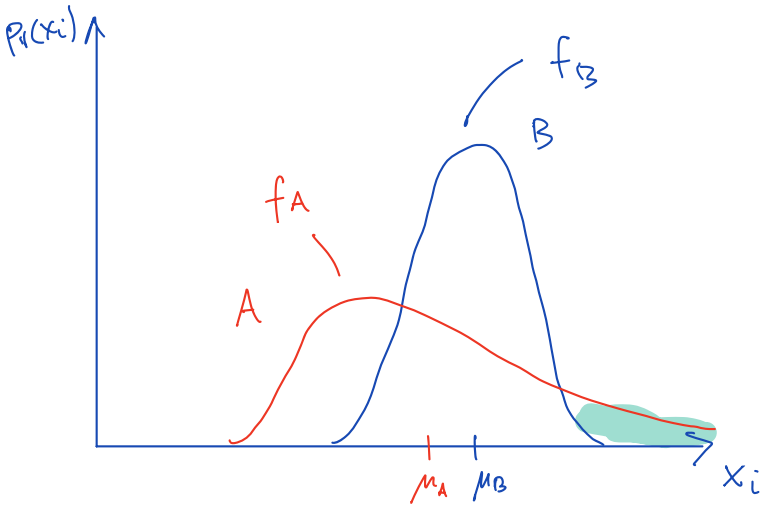


Risk averse → B

Risk neutral → B

Risk loving → B

⇒ Stochastic dominance (FO)

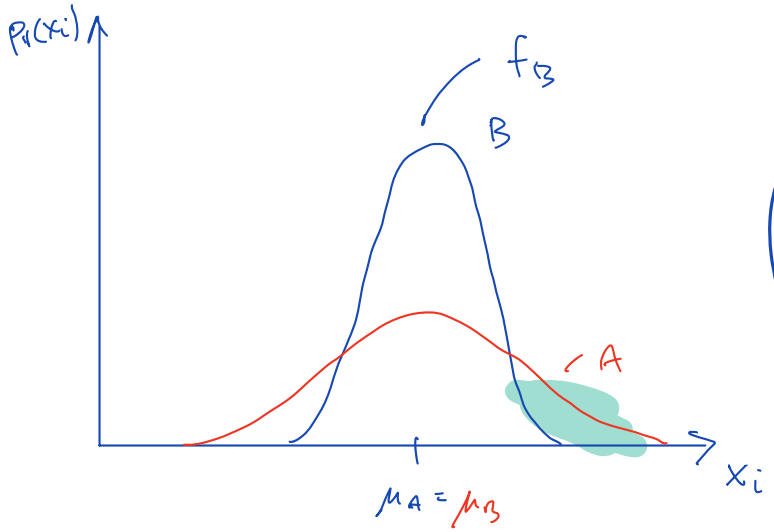


Risk averse → B

Risk neutral → B

Risk loving → ?

⇒ Stochastic Dominance (SO)

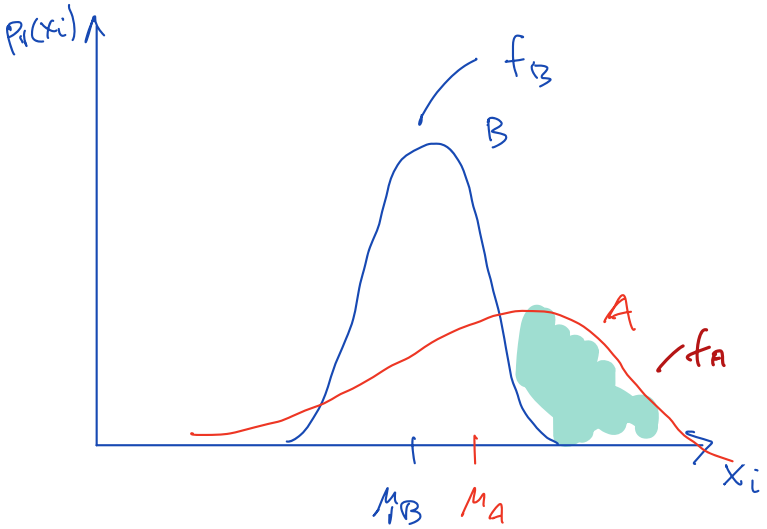


Risk averse → B

Risk neutral → A ~ B

Risk lovers → A

⇒ Stochastic Dominance (SD)



Risk averse → ?

Risk neutral → A

Risk lovers → A

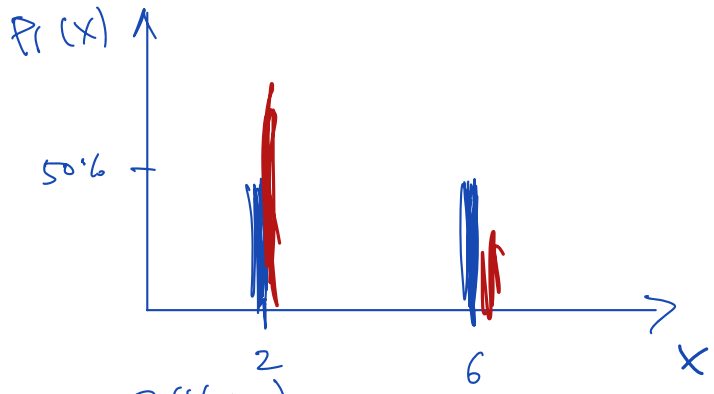
⇒

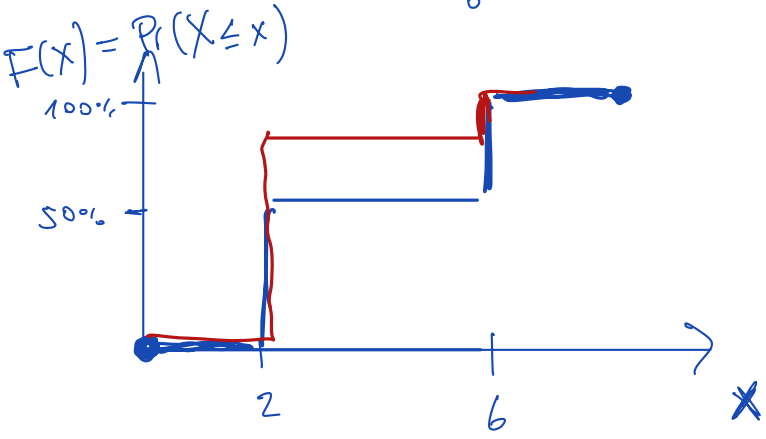
$$cdf = F(x)$$

Refresher: Cumulative Distribution Functions (cdf)

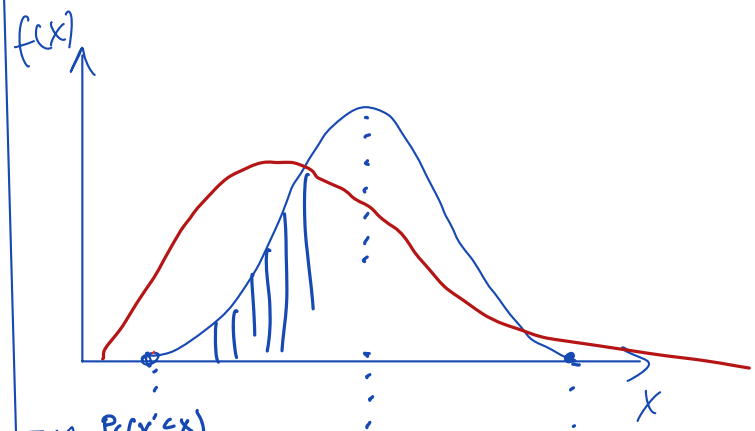
$$F(x) = Pr(X \leq x)$$

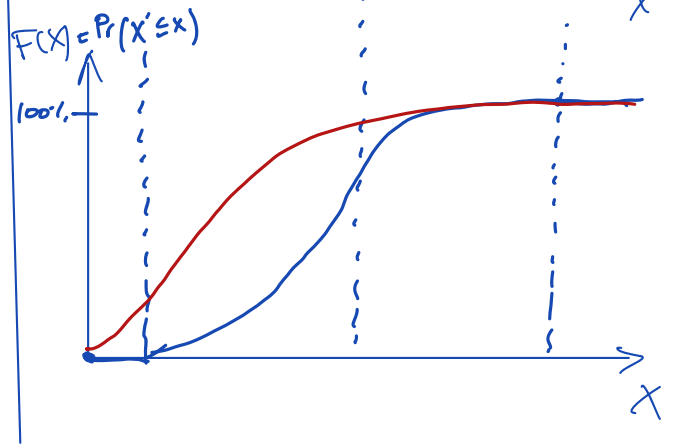
Discrete Probability Distribution





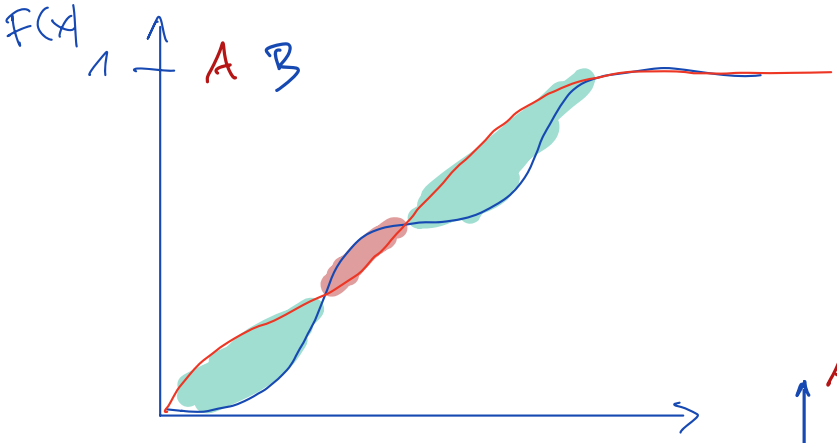
Continuous Probability Distribution

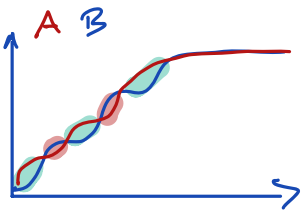


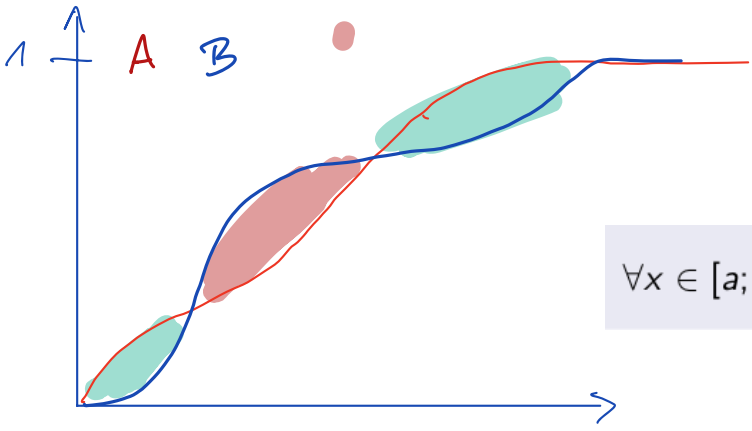


More on SOSD

Do we have SOSD here?







$$\forall x \in [a;b] : \int_a^b F_B(x)dx \leq \int_a^b F_A(x)dx$$

Exercise 1.2

A: $: 100 MU
🏠: 80 MU ← 10% chance of fire

a) A: $L_A = (0.1, 0.9; 100, 180)$

B: $L_B = (0.01, 0.18, 0.81; 100, 140, 180)$

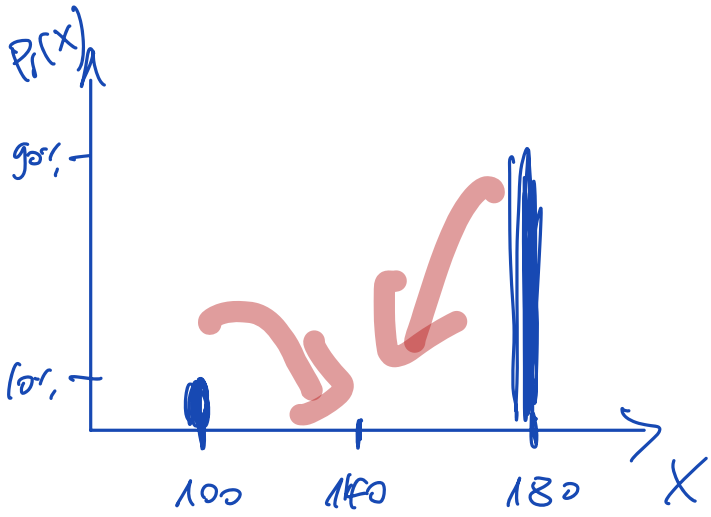
	1	2	Pr	X
Best Case	✓	✓	$0.9 \cdot 0.9 = 0.81$	180
Intermediate Case	✓	✗	$0.9 \cdot 0.1 = 0.09$	140
	✗	✓	$0.1 \cdot 0.9 = 0.09$	140
Worst Case	✗	✗	$0.1 \cdot 0.1 = 0.01$	100

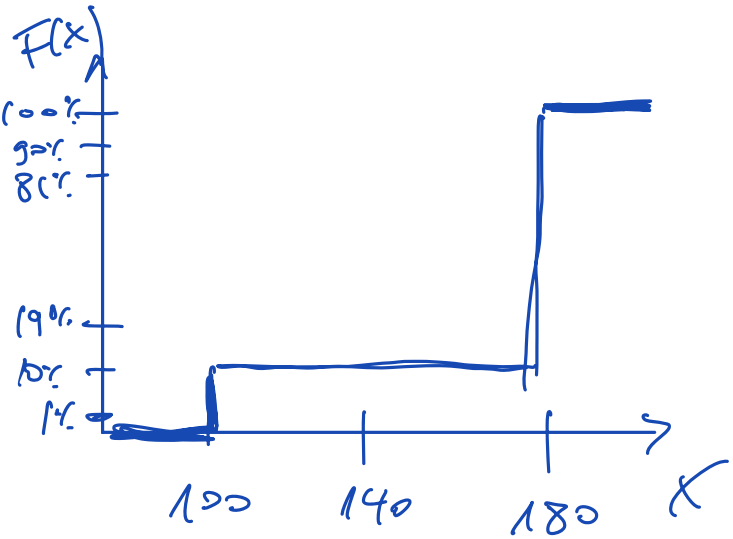
Multiplication Rule

$E(L_A) = 0.1 \cdot 100 + 0.9 \cdot 180 = 172$

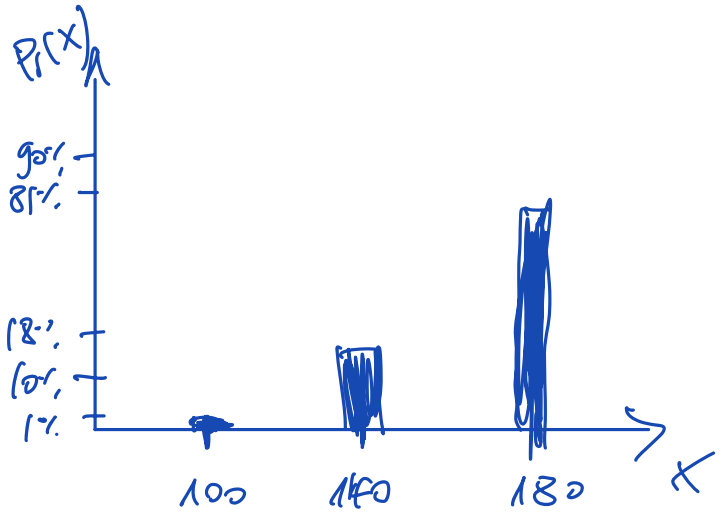
$E(L_B) = 0.01 \cdot 100 + 0.18 \cdot 140 + 0.81 \cdot 180$
$= 172$

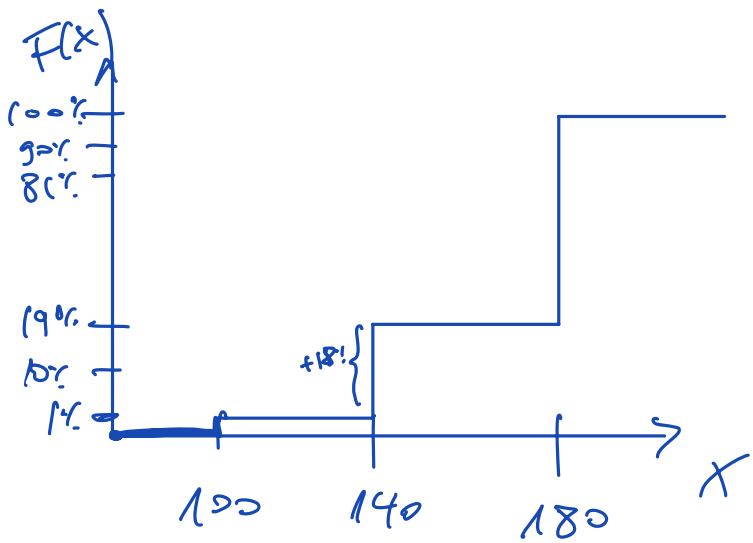
A


P(X)
100
140
180
X


F(X)
100
140
180
X

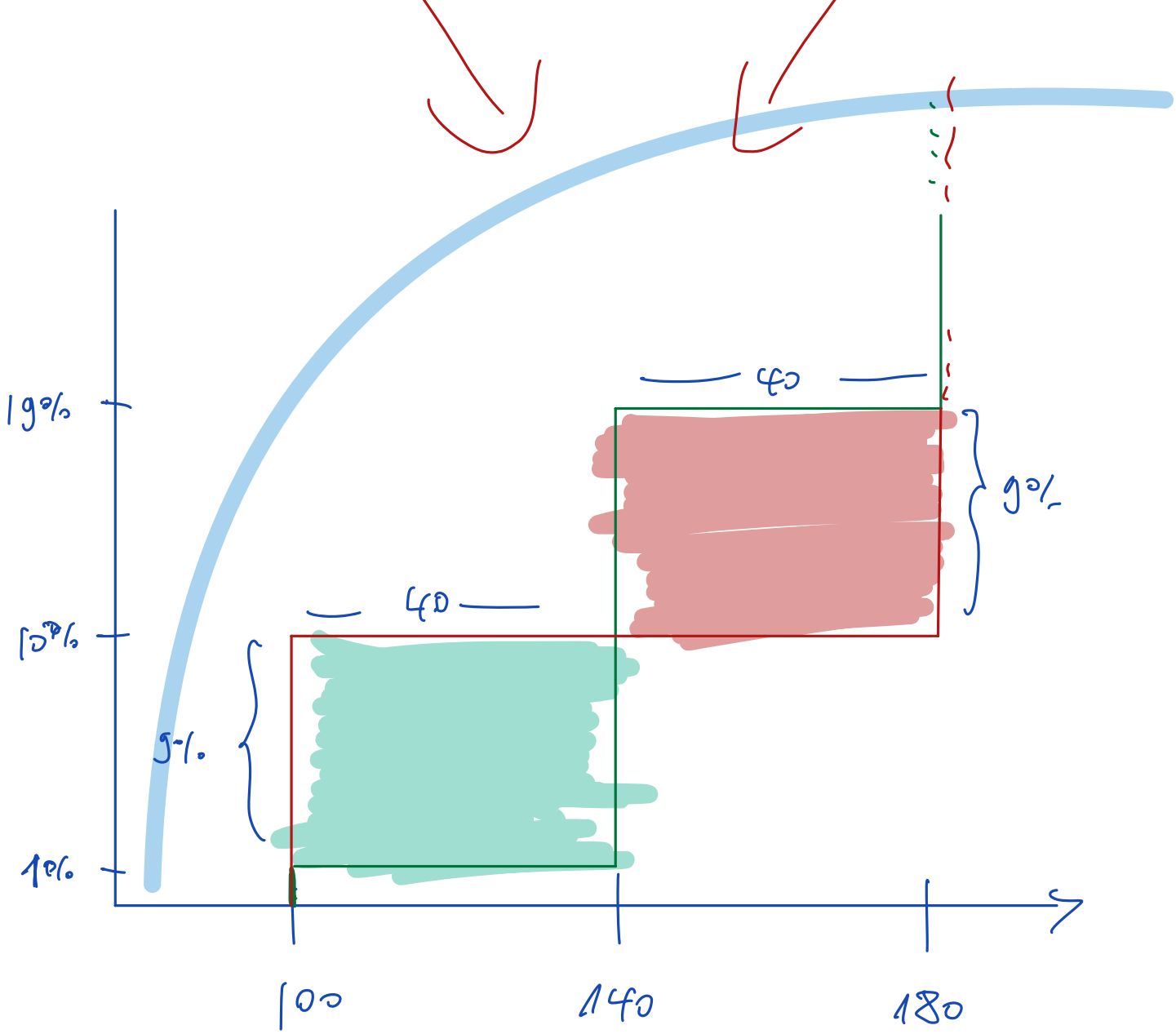
B


P(X)
100
140
180
X


F(X)
+18%
100
140
180
X

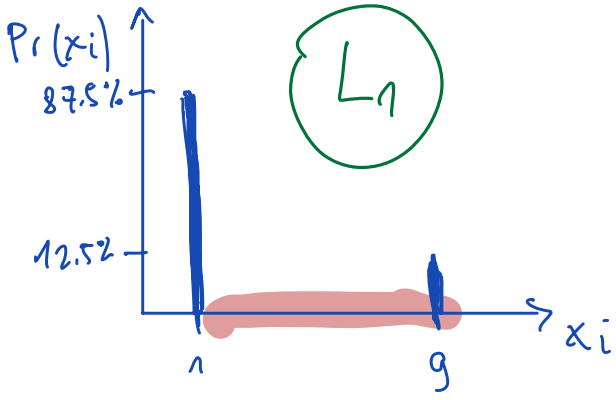
A

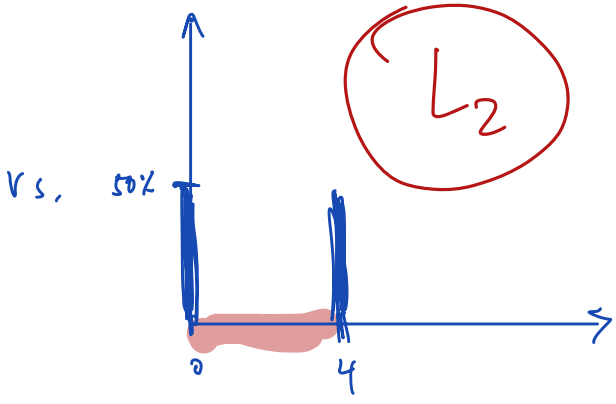
B

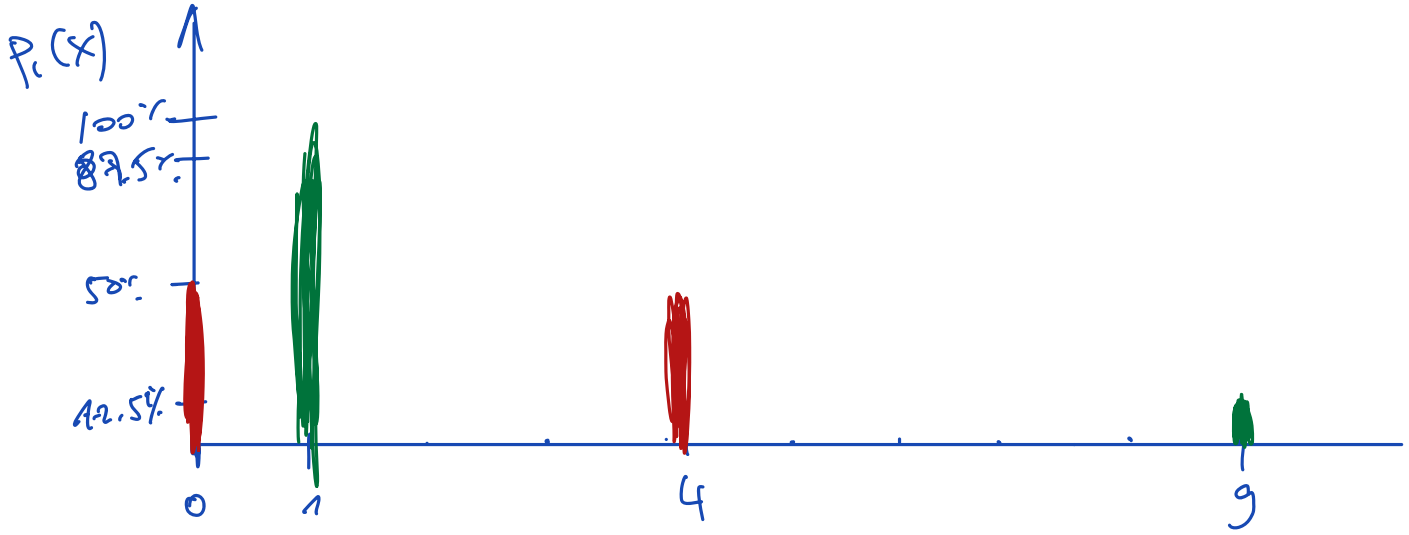

40
40
9%
9%
10%
100
140
180

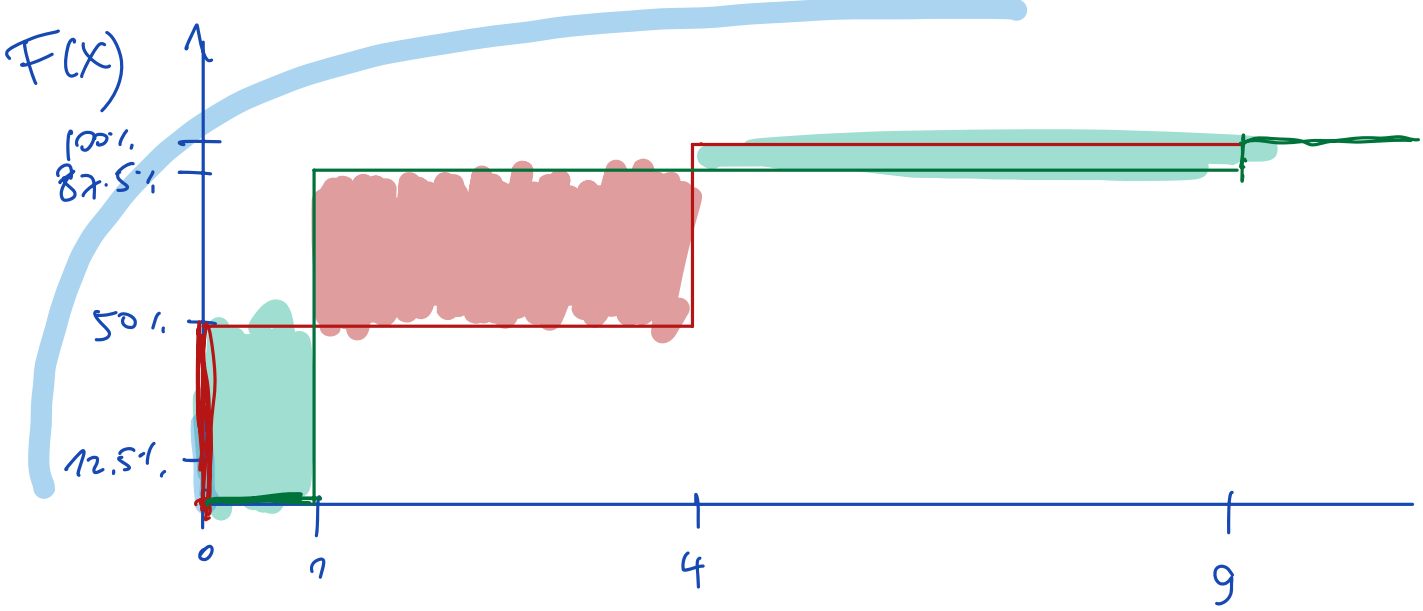
Exercise 1.3

$L_1 = (\frac{7}{8}, \frac{1}{8}; 1, 9)$ vs. $L_2 = (\frac{1}{2}, \frac{1}{2}; 0, 4)$









$u = \sqrt{x}$

↳ Choose L_1

$Eu(L_A) = \frac{10}{8}$ ⇐

$Eu(L_2) = 1$

$u = \sqrt{10+x}$

↳ Choose L_2

$Eu(L_A) = 3.447$

$Eu(L_2) = 3.452$ ⇐

Foundations
Applications
} of Risk

Refresher: Properties of $E(\cdot)$, $Var(\cdot)$

Expected Value: $E(\cdot)$

Linearity:

- $E[c \cdot \tilde{x}] = c \cdot E(\tilde{x})$
- $E[\tilde{x} + c] = E(\tilde{x}) + E(c) = E(\tilde{x}) + c$
- $E[\tilde{x} + \tilde{y}] = E(\tilde{x}) + E(\tilde{y})$

Non-multiplicativity:

- $E[\tilde{x} \cdot \tilde{y}] = E(\tilde{x}) \cdot E(\tilde{y}) + Cov(\tilde{x}, \tilde{y})$

$= 0$ if $\tilde{x}$ and $\tilde{y}$ are independent

Variance: $Var(\cdot)$

- $Var(\tilde{x} + c) = Var(\tilde{x})$

because $Var(c) = 0$

Exercise 2.1

(a) Lottery: L

- $p \rightarrow (i, \tilde{x})$
- $(1-p) \rightarrow (i, \tilde{x})$

Wealth

- $m(1+i) + a(1+\tilde{x})$
- $m(1+i) + a(1+\tilde{x})$

$$\max_{a,m} EU = p \cdot u\left(m(1+i) + a(1+\tilde{x})\right) + (1-p)\, u\left(m(1+i) + a(1+\tilde{x})\right)$$

2 extra steps: See lecture

s.t. $m + a \leq W$

$\hookrightarrow m \leq W - a$

$$\max_a EU = p \cdot \ln\left[W(1+i) + a(\bar{X} - i)\right] + (1-p)\ln\left[W(1+i) + a(\underline{X} - i)\right]$$

$\ln[W(a)]$ → Chain Rule

$$\text{FOC}: \quad \frac{\partial EU}{\partial a} = p \cdot \frac{\bar{X} - i}{W(1+i) + a(\bar{X} - i)} + (1-p) \cdot \frac{\underline{X} - i}{W(1+i) + a(\underline{X} - i)} = 0$$

$E(MB)$ $\qquad$ $E(MC)$

(b) Solve for a^*

↓ + steps missing [See upload]

$$a^* = \frac{W(1+i) - (\mu - i)}{(\bar{X} - i)\cdot(\underline{X} - i)}$$

≥ 0

(c)

> 0 if $\mu > i$

$= 0$ if $\mu = i$

< 0 if $\mu < i$

↳ Measures the volatility of a "riskiness"

(d) $W \uparrow$

$$\frac{da^*}{dW} \begin{cases} > 0 & \text{if DARA} \\ = 0 & \text{if CARA} \\ < 0 & \text{if IARA} \end{cases}$$

Detour: $u(x) \Rightarrow$ Risk-preferences

$$\boxed{A(w) = -\frac{u''(x)}{u'(x)}}$$

$$u'(x) = \frac{1}{x} = x^{-1}$$

$$u''(x) = \left(x^{-1}\right)' = -x^{-2} = -\frac{1}{x^2}$$

$$\hookrightarrow A(x) = -\frac{-\frac{1}{x^2}}{\frac{1}{x}} = +\frac{1}{x}$$

$$A(x) = \frac{1}{x} \downarrow > 0$$

$$\frac{dA(x)}{dx} = \left[\frac{1}{x}\right]' = -\frac{1}{x^2} < 0$$

$\Rightarrow$ DARA

RRA

$$\boxed{R(x) = x \cdot A(x)}$$

$$= x \cdot \frac{1}{x} = 1 = R(x)$$

$\Rightarrow$ CRRA

$$\frac{da^*}{dw} = \left[-\frac{w(1+i)(\mu-i)}{(\bar{x}-i)(\underline{x}-i)}\right]'$$

$$= -\frac{(1+i)(\mu-i)}{(\bar{x}-i)(\underline{x}-i)} > 0$$

$$\frac{da^*}{dw} > 0 \quad \text{Reason: DARA}$$

$$\frac{d\left(\frac{a^*}{w}\right)}{dw} = \left[\frac{w \cdot (1+i)(\mu-i)}{(\bar{x}-i)(\underline{x}-i) \cdot w}\right]'$$

$= 0$ (Constr)

$$\frac{d\frac{a^*}{w}}{dw} = 0 \quad \text{Reason: CRRA}$$

Big picture intuition Chapter 6

Does risk matter for firm behaviour?

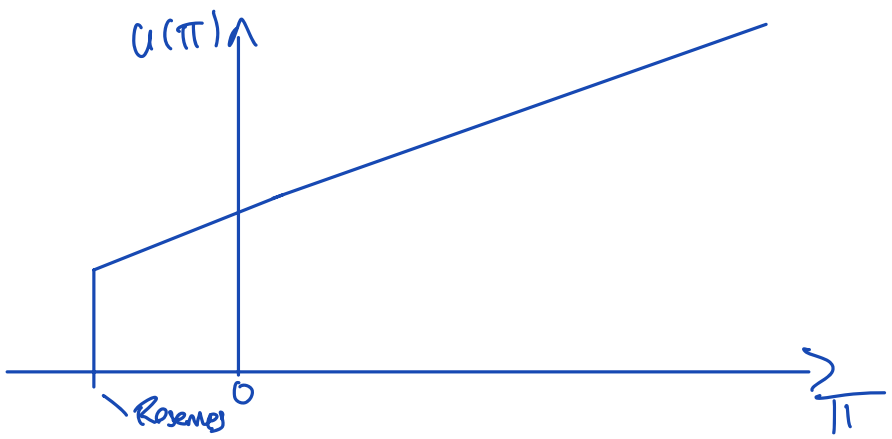
↳ FOC under Certainty: $\boxed{P = MC}$ (or $MR = MC$)

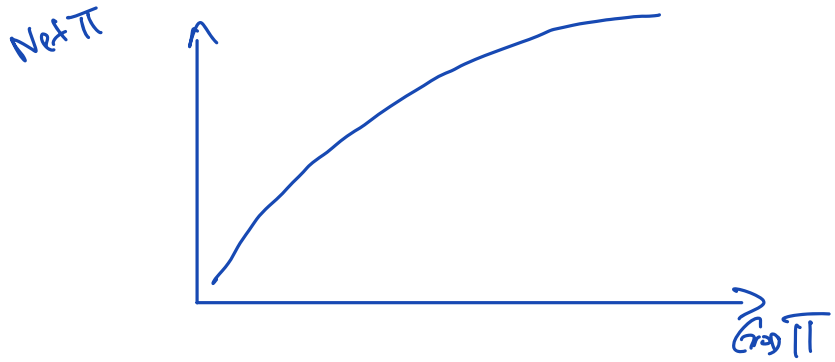
↳ x_c^*

↳ FOC under Uncertainty: $\boxed{E(P) = E(MC)}$

If risk neutral: ↳ $x_N^* = x_C^*$

If risk-averse: → $x_u^* < x_C^*$




Net π
Gross π