DAY 3

Refresher: Properties of $E(\cdot)$, $Var(\cdot)$

Expected Value: $E(\cdot)$

Linearity

- $E[c \cdot \tilde{X}] = c \cdot E(\tilde{X})$
- $E[\tilde{X} + c] = E(\tilde{X}) + E(c) = E(\tilde{X}) + c$
- $E[\tilde{X} + \tilde{Y}] = E(\tilde{X}) + E(\tilde{Y})$

Non-multiplicativity

- $E[\tilde{X} \cdot \tilde{Y}] = E(\tilde{X}) \cdot E(\tilde{Y}) + Cov(\tilde{X}, \tilde{Y})$

$= 0$ if $\tilde{X}$ and $\tilde{Y}$ are independent

Variance: $Var(\cdot)$

- $Var(\tilde{X} + c) = Var(\tilde{X})$

because $Var(c) = 0$

"Something"

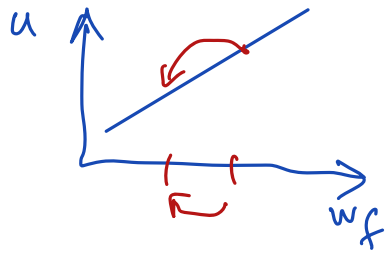
$$\mathrm{E}[\tilde{p}] = c'(a) - \left[\frac{\mathrm{Cov}(u'(\tilde{w}_f), \tilde{p})}{\mathrm{E}[u'(\tilde{w}_f)]} \right] \oplus$$

$= 0$ if risk-neutral
< 0 if risk-averse

If firm is risk-neutral

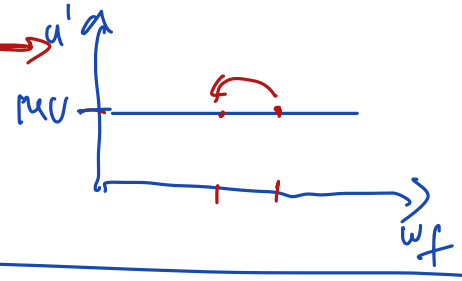
$\tilde{p} \downarrow \rightarrow \tilde{w}_f \downarrow$

$u \downarrow$



$u' \rightarrow u'$

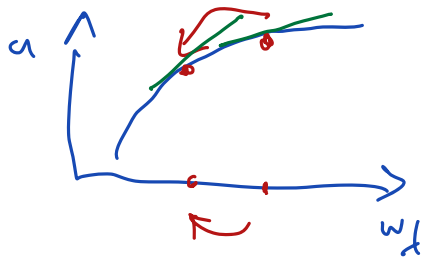
$\hookrightarrow \mathrm{Cov}(u'(\cdot), \tilde{p}) = 0$



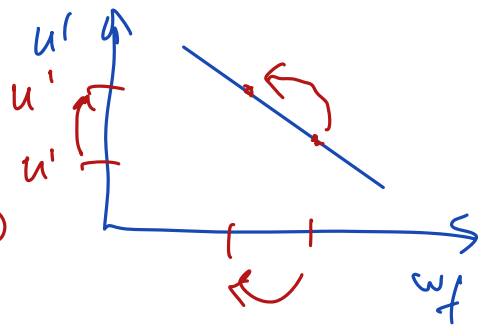
If firm is risk-averse

$\tilde{p} \downarrow \rightarrow \tilde{w}_f \downarrow$

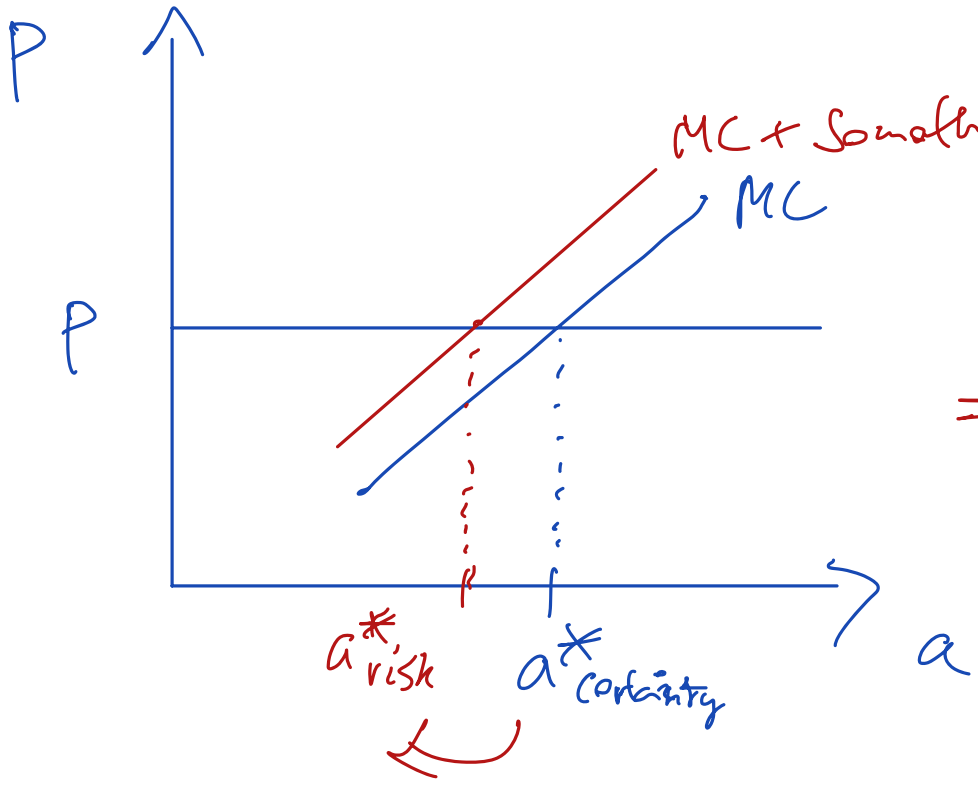
$u \downarrow$



$u' \uparrow$



$\hookrightarrow \mathrm{Cov}(u'(\cdot), \tilde{p}) < 0$



⟹ Optimal response to risk for risk-averse firm is to produce less

Exercise 2.2

$= P_g$ $= MC$ $\rightarrow a_G^* = 20$

$$\Pi^G = 100 + 9a - 5a - F = 100 + 4a - F$$

$$\Pi^B = 100 + 3a - 5a - f = 100 - 2a - F$$ → Always a loss

$\hookrightarrow a_B^* = 0$

(a)

$$\max_a EU = \frac{1}{2} \cdot \underbrace{\ln}_{(c)} \left(\underbrace{100 + 4a - F}_{wf} \right) + \frac{1}{2} \underbrace{\ln}_{(c)} \left(100 - 2a - f \right)$$

3 -1

Benchmark: Risk-neutral firm (no $\ln(\cdot)$)

$$\frac{dEU}{da} = \frac{1}{2} \cdot 4 + \frac{1}{2}(-2) = 0$$

$$= 2 - 1 = 1 \neq 0$$ ← Reduce as much as you can

$\hookrightarrow a^* = 20$

Risk-averse firm

$$\frac{dEu}{da} = \frac{1}{2} \frac{4}{(100 + 4a - F)} + \frac{1}{2} \frac{-2}{(100 - 2a - F)} = 0$$

$$\frac{100+4a-F}{2} = 100-2a-F \quad \Big| \text{ Solve for } a$$

$$50+2a-\frac{F}{2} = 100-2a-F$$

$$4a = 50-\frac{F}{2} \quad \Big| \cdot 4$$

$$\boxed{a^* = \frac{50}{4} - \frac{F}{8}} \qquad F=2$$

$$a^* = 12.25 < 20$$

↑ Benchmark risk-neutral firm

(b) Why do fixed cost matter if firm is risk-averse

F ↑ → makes firm poorer → DARA → Risk aversion goes up

q^* should go down

$$q^* = \frac{50}{4} \cdot \frac{F}{8}$$

$F = 3$

$$q^* = 12.125$$

(before it was 12.25)

(c) Risk changes

ν → 9 → 8

ν → 3 → 4

$\hat{=}$ Reduction in risk

↓

a^* should go up
Because producing may become less risky

↓

New FOC $\quad \frac{1}{2} \cdot \frac{3}{100+3a-F} + \frac{1}{2} \frac{-1}{100-a-F} = 0$

↓ 3 Steps (same as above)

$$\boxed{a^* = \frac{1}{3}(100 - F)}$$

$F = 2$

$$= \frac{98}{3} \doteq 32.\overline{6}$$

↳ $a^* = 20$

↳ If $p = 6$ (no risk)

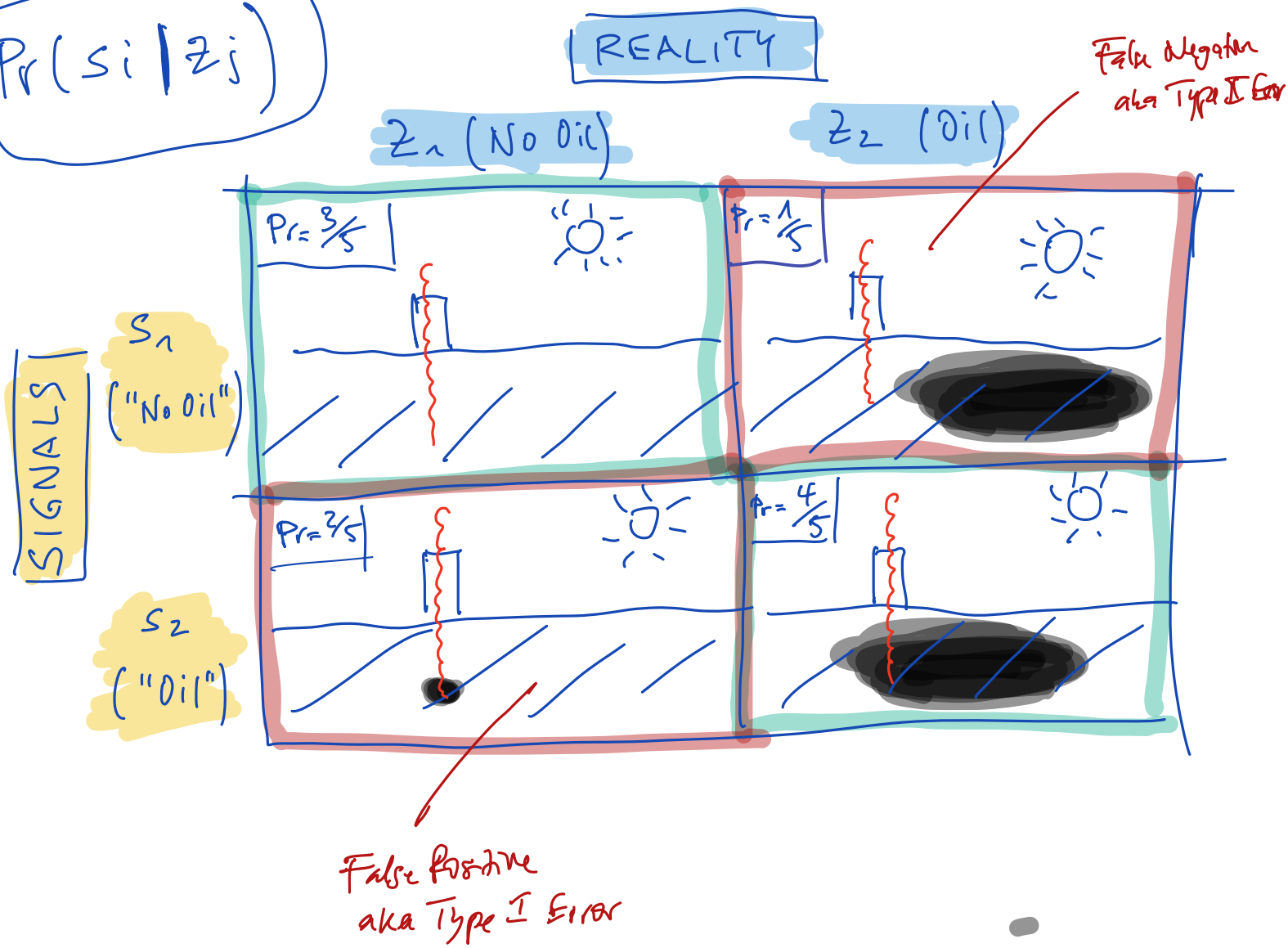
↳ $P > MC$

$6 > 5 \Rightarrow a^* = \infty$

↳ $a^* = 20$

Illustration: Signals ↔ Reality

$Pr(s_i | z_j)$



3 Ways to calculate $Pr(z_i|s_k)$

1 Hardest : Long Version of Bayes' Theorem ← Lecture

When? If I only give you
- $Pr(z_i)$
- $Pr(z_i|s_k)$

$$Pr(z_i|s_k) = \frac{Pr(s_k|z_i) \cdot Pr(z_i)}{Pr(s_k|z_i) \cdot Pr(z_i) + Pr(s_k|\bar{z}_i) \cdot Pr(\bar{z}_i)}$$

$Pr(s_k|\bar{z}_i) \cdot Pr(\bar{z}_i) = Pr(s_k)$

Need to calculate $Pr(s_k)$ yourself

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2 Intermediate : Short Version of Bayes' Theorem

When? If I give you
- $Pr(z_i)$
- $Pr(z_i|s_k)$
- $Pr(s_k)$

$$Pr(z_i|s_k) = \frac{Pr(s_k|z_i) \cdot Pr(z_i)}{Pr(s_k)}$$

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3 Easiest : No Bayes' Theorem ← Exercise 2.3

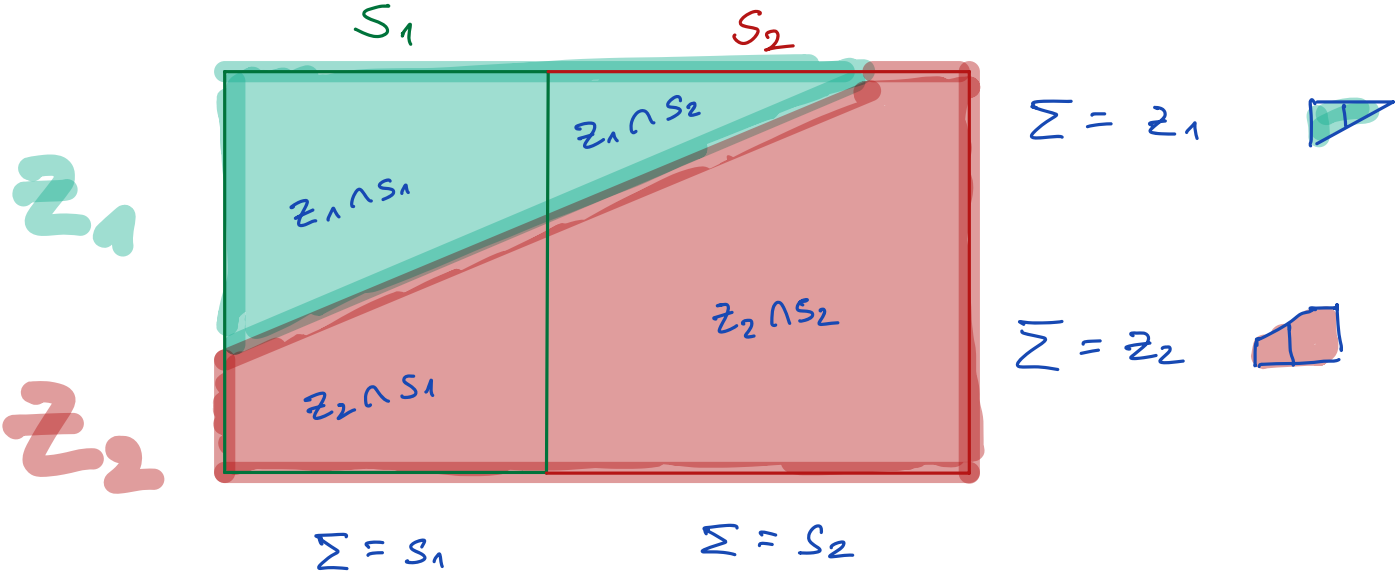
When? If I give you
- $Pr(z_i \cap s_k)$

You can calculate $Pr(z_i)$ and $Pr(s_k)$ yourself

$$Pr(z_i|s_k) = \frac{Pr(z_i \cap s_k)}{Pr(s_k)}$$

From Definition of Joint Probability

Venn Diagram



Exercise 2.3

- 2 States of World
  - z_1 = Good Weather
  - z_2 = Bad Weather

- 2 Signals
  - s_1 = "Sun" ($\hat{=}$ Good Weather)
  - s_2 = "Rain" ($\hat{=}$ Bad Weather)

Decision Problem:

- a_1 = Dry Tyres
- a_2 = Wet Tyres

Payout Matrix:

	z_1	z_2
a_1	45,000	0
a_2	20,000	20,000

(a) Find Ex-post probabilities:

Wanted: $Pr(z_i | s_k)$

Given: $Pr(z_i \cap s_k)$

	s_1	s_2	Σ
z_1	0,2	0,1	0.3 = $Pr(z_1)$
z_2	0,2	0,5	0.7 = $Pr(z_2)$
Σ	0.4 = $Pr(s_1)$	0.6 = $Pr(s_2)$	

$$Pr(z_i | s_k) = \frac{Pr(z_i \cap s_k)}{Pr(s_k)}$$

Definition of a Conditional Probability

$$Pr(z_1 | s_1) = \frac{0.2}{0.4} = \frac{1}{2}$$

$$Pr(z_1 | s_2) = \frac{0.1}{0.6} = \frac{1}{6}$$

$$\Pr(z_2|s_1) = \frac{0.2}{0.4} = \frac{1}{2}$$

$$\Pr(z_2|s_2) = \frac{0.5}{0.6} = \frac{5}{6}$$

↳ $\Pr(z_i|s_u)$

	s_1	s_2
z_1	$\frac{1}{2}$	$\frac{1}{6}$
z_2	$\frac{1}{2}$	$\frac{5}{6}$
Σ	1	1

Show that w/o Signal, Veiled fits wet fly (C)

Payout Matrix:

	z_1	z_2
a_1	45,000	0
a_2	20,000	20,000

- a_1 = Dry Tyres
- a_2 = Wet Tyres

What would be optimal w/o signal?

If a_1: $Eu(a_1) = P_r(z_1) \cdot 45{,}000 + P_r(z_2) \cdot 0$

$P_r(z_1) = 0.3$, $P_r(z_2) = 0.7$

If a_2: $Eu(a_2) = 20{,}000 \approx 13{,}500$

⇒ Wet Tyres if no signal

(c) With Signal

If s_1 (Good Weather forecast)

$P_r(z_1 | s_1) = \frac{1}{2}$

$P_r(z_2 | s_1) = \frac{1}{2}$

a_1: $Eu(a_1) = \frac{1}{2} \cdot 45{,}000 + \frac{1}{2} \cdot 0$

$= 22{,}500$

a_2 $Eu(a_2) = 20,000$ ✓

If s_2 (Bad Weather forecast)

$Pr(z_1 | s_2) = 1/6$

$Pr(z_2 | s_2) = 5/6$

a_1: $Eu(a_1) = \frac{1}{6} 45,000 + \frac{5}{6} \cdot 0$

$= 7,500$

a_2 $Eu(a_2) = 20,000$

EU Signal $= Pr(s_1) \cdot 22,500 + Pr(s_2) \cdot 20,000$

$= 0.4 \cdot 22,500 + 0.6 \cdot 20,000$

$= 21,000$

$$\text{Value} = EU^{\text{Signal}} - EU^{\text{No Signal}}$$

$$= 21{,}000 - 20{,}000$$

$$= 1{,}000 < 2{,}000$$

Value of Signal

Price of Signal

$\therefore$ Red Bull will NOT buy the signal