Economic Foundations and Applications of Risk (2024)

Exam

November 8th, 2024

Instructions

- You are supposed to **answer all questions**.
- You have **60 minutes for a total of 60 points.**
- **Show how you derived your answers** (either by calculation or by verbal reasoning).
- Your answers can be in **German or English** (or any combination that I am able to understand).
- **You may use** a (non-programmable) **calculator**.
- **Good luck!!!**

1. **Risk preferences [35 Points]**

   (a) Consider **Individual 1** with the following utility function, where y stands for income: $u_1 = \ln(y)$. Calculate Individual 1's Coefficients of Absolute and Relative Risk Aversion, $A_1(y)$ and $R_1(y)$. *(Hint: Should you not trust your own calculations, you can use this alternative result for the follow-up questions below:* $A_1^{alt}(y) = \frac{1}{2y}$*)* [5 Points]

   (b) Calculate how Individual 1's absolute and relative aversion change as she gets richer. Are the results empirically realistic? [5 Points]

   (c) Now consider **Individual 2** with the following utility function, where y again stands for income: $u_2 = y^{\frac{1}{2}}$. Calculate Individual 2's Coefficients of Absolute and Relative Risk Aversion, $A_2(y)$ and $R_2(y)$. *(Hint: Should you not trust your own calculations, you can use this alternative result for the follow-up questions below:* $A_2^{alt}(y) = \frac{1}{4y}$*)* [5 Points]

   (d) Calculate how Individual 2's absolute and relative aversion change as she gets richer. Are the results empirically realistic? [4 Points]

   (e) Which of the two individuals is more risk-averse towards a gamble over absolute amounts of money if both have an income of $y = 100$? Is that individual also **globally** more risk-averse than the other? [4 Points]

   (f) Draw the indifference curves of Individual 1 and Individual 2 into a 2-States-of-the-World Diagram and depict which of the two has the higher certainty equivalent for the following lottery: $L = (\frac{1}{2}, \frac{1}{2}; 8, 2)$ *(Hint: A rough sketch is sufficient, and you do* ***not*** *need to calculate the values of the certainty equivalents. Also, you do* ***not*** *have to factor in the income of* $y = 100$ *from question (e).)* [7 Points]

   (g) Draw the risk premiums of Individual 1 and Individual 2 for this lottery into the diagram from question (f). *(Hint: To graphically find the risk premium, draw the indifference curve of Individual 3 with the following utility function into the diagram:* $u_3 = y$*. This will provide you with the benchmark you need.)* [5 Points]

2. **Production under uncertainty & the value of information [25 Points]**

Consider a **risk-neutral** firm and assume that it has sufficient reserves to cover any short-term losses. The firm produces a commodity, a, with marginal cost of $c = 6$. Assume that there are no fixed costs. The firm's maximum capacity is $\bar{a} = 50$. Under unfavourable market conditions (with probability 50%), the market price of the commodity is $p_B = 2$, and otherwise $p_G = 8$. The firm needs to make its production decision before market prices are known.

(a) Write down the expected-profit problem of the firm, and determine the optimal output a^* it produces as well as the expected profit it makes when facing this price uncertainty. [7 POINTS]

The firm has the option to do market research to improve its forecast of the uncertain market price. If the forecast is positive (s_G), the **ex-post** probabilities are 80% for $p_G = 8$, and 20% for $p_B = 2$. If the forecast is negative (s_B), the **ex-post** probabilities are 20% for $p_G = 8$, and 80% for $p_B = 2$. Assume the firm also knew that the probability of a positive forecast was $\Pr(s_G) = 50\%$ and $\Pr(s_B) = 50\%$ for a negative forecast. The resulting matrix of **ex-post** probabilities, $\Pr(p_i|s_j)$, is therefore given as:

	s_G	s_B
p_G	0.8	0.2
p_B	0.2	0.8

*(Hint: Compared to the cases discussed during the guest course, this problem is easier because you do **not** need to calculate the ex-post probabilities. These are already given to you in the matrix above.)*

(b) Determine the optimal output a^* and the resulting expected profit if the firm did market research and received a negative forecast, s_B. [5 POINTS]

(c) Determine the optimal output a^* and calculate the resulting expected profit if the firm did market research and received a positive forecast, s_G. [6 POINTS]

(d) By combining your results from (a), (b), and (c), calculate the owner's maximum willingness to pay for doing the market research. [7 POINTS]